Research plan

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"Formulation of quantum field theory by gradient flow and its numerical analysis"

Abstract: We will approach the themes of spontaneous symmetry breaking and supersymmetry using the gradient flow (GF) method, and further use this technique to develop a numerical analysis method using the stochastic quantization.

1. Spontaneous symmetry breaking and gradient flow

Investigating the phase structure of spontaneous gauge symmetry breaking is accomplished by defining the correct order parameter. The natural definition of the gauge-invariant order parameter is the Higgs two-point correlation function, however, we cannot define it due to the divergence. We can remove the divergence and define the new order parameter using the GF method. In this study, we develop a new analytical method to investigate the phase structure by defining a new order parameter for spontaneous symmetry breaking using the flow Higgs field.

2. Supersymmetric gradient flow and quantum field theory

The gauge symmetry plays an important role in the UV finiteness in the GF for SU(N) Yang-Mills theory. In a theory that includes interactions other than gauge interactions, whether the gradient flow possesses the property of UV finiteness is, in general, nontrivial. On the other hand, since SUSY is an exceptionally powerful symmetry, it is possible for the theory to possess the UV finiteness even in the presence of non-gauge interactions.
We showed that GF methods applied to the Wess-Zumino model yield UV finiteness to all orders in perturbation, provided a non-renormalization theorem and suitable initial conditions. This model achieves UV finiteness through a mechanism that is fundamentally different from that of gauge theories, serving as an example of how GF can be effective even in theories without gauge symmetry. Analyzing this theory provides insights into the physical principles underlying the unique finiteness property of GF and what it depends on. Furthermore, it opens the door to applications in exact renormalization group methods with explicit supersymmetry and in numerical computations.

3. Numerical approach to stochastic quantization

As a development of analytical methods for non-perturbative field theories, we study the Parisi-Wu stochastic quantization. The Langevin equation used in stochastic quantization is written in the same form as the gradient flow equation. In constructing the Langevin equation, the method of the gradient flow equation with keeping symmetries can be applied. As a numerical approach, we develop a Python code to solve the Langevin equation and measure physical quantities.