2. future research

Our future research program will extend our ongoing work in geometric analysis and the geometry of submanifolds.

2.1. **Rigidity Gradient Ricci Solitons.** We would like to make progress on Cao's conjecture, which proposes that a complete gradient shrinking Ricci soliton with constant scalar curvature must be rigid.

- Remove Technical Assumptions: we will develop new analytic techniques to eliminate the bounded curvature assumption in the $R = 2\lambda$ case, aiming for a complete classification of five-dimensional solitons with constant scalar curvature.
- Higher Dimensions and New Obstructions: For dimensions $n \geq 6$, we plan to give a complete classification. A key strategy will be to analyze the interplay between the potential function f and the Weyl tensor of its levelsets.
- Sasaki-Ricci solitons: we will study five-dimensional Sasaki-Ricci solitons since Li-Wang classified gradient shrinking Kähler-Ricci solitons of complex dimension two (Acta Math.2025).

2.2. **Geometry of Submanifolds.**
- Stability of Minimal Submanifolds and Bernstein-Type Problems: Inspired by my participation in the NSFC project led by Prof. Cheng, we will delve into the stability theory for minimal submanifolds.
- Conformal Geometry of Submanifolds: we will study the conformal geometry VM-hypersurfaces, and the classification of Willmore surfaces with closed Möbius form.

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