FENGJIANG LI'S RESEARCH STATEMENT

1. RESEARCH ACHIEVEMENTS

My research interests lie in differential geometry and geometric analysis, with a particular focus on gradient Ricci solitons, geometric flows, and the conformal geometry of submanifolds.

1.1. **Rigidity and Classification of Gradient Ricci Solitons.** Gradient Ricci solitons, satisfying $Ric + \nabla^2 f = \lambda g$, are critical in the study of the Ricci flow and its singularities.

In 2019, Prof. Huai-Dong Cao raised the following conjecture: a complete gradient shrinking soliton with constant scalar curvature must be rigid, i.e., a finite quotient of $N^k \times \mathbb{R}^{n-k}$ for some Einstein manifold N of positive scalar curvature. We focus our attention on five-dimensional gradient shrinking Ricci solitons with constant scalar curvature. Fernández-López and García-Río (Proc. Amer. Math. Soc., 2016) proved that their scalar curvature $R \in \{0, 2\lambda, 3\lambda, 4\lambda, 5\lambda\}$. Moreover, if $R = 0$ or 5λ, they are Einstein (Petersen-Wylie, 2009); if $R = 4\lambda$, they are finite quotient of $N^4 \times \mathbb{R}$, where N^4 is a four-dimensional Einstein manifold.

- We proved that it is a finite quotient of $\mathbb{R}^2 \times \mathbb{S}^3$ if $R = 3\lambda$ (arXiv:2411.10712); for $R = 2\lambda$, it is isometric to a finite quotient of $\mathbb{R}^2 \times \mathbb{S}^3$ under the additional condition of bounded curvature (arXiv: 2506.00887).
- We have also studied Sasaki-Ricci solitons and proved that in low dimensions, any such soliton with constant scalar curvature must be Sasaki-Einstein (arXiv: 2406.16430, 2502.16148).
- Our work extends to problems in geometric analysis, often involving curvature conditions and functional monotonicity (Pacific J. Math., 2025, Results Math., 2021, Proc. Amer. Math. Soc., 2020).

1.2. **Conformal Geometry of Submanifolds.** Another portion of my research concerns the Möbius and Laguerre geometry of submanifolds (Math. Anal. Appl., 2025, Math. Nachr., 2020, Intern. J. Math. 2016, Acta. Math. Sin. (Engl. Ser.), 2015).

- We introduced and systematically studied a new class of surfaces with closed Möbius form. We showed that a surface with closed Möbius form can be determined by a smooth function satisfying a fifth-order PDE and provided a complete classification of isothermic surfaces within this class (Differ. Geom. Appl., 2015).
- We linked the sharpness of Huisken's inequality ($|\nabla h|^2 \geq \frac{3n^2}{n+2}|\nabla H|^2$) to the study of VM-hypersurfaces (those with vanishing minimal norm tensor of ∇h). We proved that any VM-hypersurface in $N^{n+1}(c)$ for $n \geq 3$ must either have a parallel second fundamental form or be a rotational hypersurface generated by a free $(\frac{3}{2}, a)$-elastic curve in a totally geodesic subspace $N^2(c)$ of $N^{n+1}(c)$.