

Research plan (Mikiya Masuda) January, 2025

I would like to regard toric topology widely as mathematics about the topology and geometry of torus actions and related combinatorics and extend and deepen this area. Currently, I am working on the following projects.

(1) Research on mathematics related to Hessenberg varieties which are subvarieties of flag varieties. Together with Hiraku Abe, Megumi Harada, Tatsuya Horiguchi, I worked on the cohomology rings of Hessenberg varieties during this decade. It turned out that this work is related to hyperplane arrangements and Stanley-Stembridge conjecture on graph theory. My goal is to solve the Stanley-Stembridge conjecture and I have been working with Takashi Sato for three years to aim the solution of the conjecture. So far, we obtained results on

- (1) Characterization of regular semisimple Hessenberg varieties with cohomology rings generated in degree 2.
- (2) Relation between the twins of regular semisimple Hessenberg variety and unicellular LLT polynomials.
- (3) Automorphism groups of regular semisimple Hessenberg varieties.

Recently, I found that the modular law for regular semisimple Hessenberg varieties can be proved in terms of GKM theory (joint work with Sato and Horiguchi). Combining our work with a result by Abreu-Nigro (the proof is also elementary), we can obtain an alternative proof Shareshian-Wachs conjecture first proved by Brosnan-Chow. Their proof uses highly advanced algebraic geometry. I am planning to deepen the research in this direction and reach the solution of Stanley-Stembridge conjecture.

(2) I have been working on torus orbit closures in flag varieties with Eunjeong Lee and Seonjeong Park for these five years and will continue this project. I am planning to extend our results to torus orbit closures in partial flag varieties.