

(2) Summary of Previous Research

Soma Ohno

I provide backgrounds and summaries of the following two peer-reviewed papers.

peer-reviewed paper [1] : S. Ohno. *Rarita-Schwinger fields on nearly parallel G_2 -manifolds*.

peer-reviewed paper [2] : S. Ohno, T. Tomihisa. *Rarita-Schwinger fields on nearly Kähler manifolds*.

Background of Previous Research

“Conventional” spin geometry is the area of differential geometry dealing with the Dirac operator and special spinor fields on the spinor bundle $S_{1/2}$. In particular, an element of the kernel of the Dirac operator is called a harmonic spinor. It is known that the existence or non-existence of this harmonic spinor is controlled by the curvature. On the other hand, “new” spin geometry deals with the Rarita-Schwinger operator and Rarita-Schwinger fields on the spin-3/2 spinor bundle $S_{3/2}$, which can be considered analogous to the Dirac operator and harmonic spinors, respectively. Unlike harmonic spinors, the existence or non-existence of Rarita-Schwinger fields depends on the geometric structure and the dimension of the manifold. Thus, the question naturally arises: “On manifolds with what geometric structure can we clarify Rarita-Schwinger fields?”. Manifolds with Killing spinors, which are spinor versions of Killing vector fields, are one of the interesting classes that answer this question.

Background of Research results

(1) Studied Rarita-Schwinger fields on nearly parallel G_2 -manifolds (peer-reviewed paper [1])

A nearly parallel G_2 -manifold is one of manifolds with Killing spinors, and is a 7-dimensional Einstein manifold with positive scalar curvature. By constructing some formulas for a “good connection on nearly parallel G_2 -manifolds”, I showed that the space of Rarita-Schwinger fields is isomorphic to a certain subspace of the eigenspace of the Laplacian on compact nearly parallel G_2 -manifolds. Only “negative” results have so far been deduced from the above. In other words, I found that there is no Rarita-Schwinger field on some concrete nearly parallel G_2 -manifolds. On the other hand, according to Y. Homma and U. Semmelmann, there are Rarita-Schwinger fields on many torsion-free G_2 -manifolds. This means that even on manifolds with the same structure group, the behavior of Rarita-Schwinger fields can be quite different.

Furthermore, I also succeeded in identifying the space of infinitesimal deformations of Killing spinors on nearly parallel G_2 -manifolds. There is a one-to-one correspondence between the space of Killing spinors and the space of nearly parallel G_2 -structures on 7-dimensional spin manifolds. Our result is therefore important as an investigation of the infinitesimal deformation of nearly parallel G_2 -structures by B. Alexandrov and U. Semmelmann through Killing spinors

(2) Studied Rarita-Schwinger fields on nearly Kähler manifolds (peer-reviewed paper [2])

A nearly Kähler manifold is one of manifolds with Killing spinors. In particular, it is an Einstein manifold with positive scalar curvature. There exists a “good connection” on nearly Kähler manifolds that have similar properties to “the good connection on nearly parallel G_2 -manifolds” described in (1). Therefore, I could construct formulas for “good connections on nearly Kähler manifolds” and showed that the space of Rarita-Schwinger fields is isomorphic to the space of harmonic 3-forms on 6-dimensional compact nearly Kähler manifolds. This result leads to the existence of a two-dimensional space of Rarita-Schwinger fields on

homogeneous and non-homogeneous $S^3 \times S^3$. On the other hand, I found that there is no Rarita-Schwinger field on $S^3 \times S^3$ equipped with the standard metric, which is not a nearly Kähler manifold. This is important because it is the first example where Rarita-Schwinger fields depend not only on the topology but also on the metric.

Furthermore, using a similar method to identify the space of Rarita-Schwinger fields, I also succeeded in identifying the space of infinitesimal deformations of Killing spinors on nearly Kähler manifolds. There is a one-to-one correspondence between the space of Killing spinors and the space of nearly Kähler structures on 6-dimensional spin manifolds. Our result is therefore important as an investigation of the infinitesimal deformation of nearly Kähler structures by A. Moroianu, P.-A. Nagy and U. Semmelmann through Killing spinors.