

About Previous Research(Hiroshige Shiga)

My research can be broadly divided into two areas. One focuses on the study of deformation spaces of Riemann surfaces and Kleinian groups, and the other on the Riemann surfaces themselves. I will explain these two areas in detail. The paper numbers refer to the attached list of publications.

Study of Deformation Spaces of Riemann Surfaces and Kleinian Groups:

A Riemann surface is a one-dimensional complex manifold, and its deformation space is considered as the space formed by deforming its complex structures. My research targets deformations of a single complex structure through quasiconformal mappings. Kleinian groups are discrete groups consisting of Möbius transformations, and Riemann surfaces can be realized as quotient spaces of Kleinian groups. Thus, deformations of Riemann surfaces via quasiconformal mappings can be interpreted as deformations of Kleinian groups via quasiconformal mappings.

The most famous example of such deformation spaces is the Teichmüller space, which is known to be a complex manifold through Ahlfors-Bers theory. I have studied the complex structures of the Teichmüller space and deformation spaces of Kleinian groups, elucidating the geometric and analytic properties of finite-dimensional Teichmüller spaces ([5], [13], [15], [22], [45]). Additionally, I have researched holomorphic mappings on and into these deformation spaces, providing analytic proofs of the Parshin-Arakelov theorem and studies on the Carathéodory metric ([11], [16], [32], [35], [40], [46]).

When the topological type of a Riemann surface becomes infinite, its Teichmüller space also becomes infinite-dimensional, showing different characteristics from the finite-dimensional case. For example, the quasiconformal mapping class group may not act discontinuously. In paper [30], we provided sufficient conditions for discontinuity based on the hyperbolic geometric properties of Riemann surfaces. In [27], I constructed examples showing that the Teichmüller distance and Length spectrum distance, which define equivalent topologies in finite-dimensional Teichmüller spaces, provide different topologies in the infinite-dimensional case, and I also provided sufficient conditions for their equivalence. Recently, I have been researching the quasiconformal deformation spaces of infinite-type Riemann surfaces obtained as complements of Cantor sets ([1], [4], [7]).

Study of Riemann Surfaces:

Compact Riemann surfaces are also algebraic curves, with well-established classical theories. On the other hand, open Riemann surfaces exhibit different properties from compact ones. They have a rich variety of holomorphic and meromorphic functions, providing a powerful stage for complex analysis; however, classical theorems such as Riemann-Roch and Abel's theorem do not hold in their original forms. In [49], I investigated the conditions under which classical theorems hold within a specific framework using extremal distances.

The unit disk in the complex plane is the most elementary open Riemann surface, with an abundance of established theorems concerning holomorphic and meromorphic functions. Hence, it serves as a model for developing complex analysis on open Riemann surfaces. Papers [10], [14], [21], [36], [37], [38], and [51] are extensions of theorems valid in the unit disk to open Riemann surfaces. In relation to "the study of deformation spaces of Riemann surfaces," I have also examined how conformal invariants are affected when the complex structure of a Riemann surface is deformed ([2], [3], [26], [34], [47], [48]). These studies yield interesting results from the perspective of deformation spaces of Riemann surfaces.