Program on my study

As I mentioned in the item "About my past research", my theme has been mainly on the Capelli identities. There are lots of different variations and possibilities to develop on it. I dare say, too many possibilities, and no one could exhaust such theme. We could feel, however, any, though small, possibilities to catch something there. Technical difficulties are in that we treat sort of complicated no-commutativities. Still, in many points, we succeeded in getting non-trivial results.

In the past, I had some hopeful ideas to new results. But, I regret I had no enough time to pursue those ideas. Now, I am hoping to return to face these ideas.

One example is the Capelli identities for the dual pair $(\mathfrak{sp}_{2n}, \mathfrak{o}_m)$. We know that such identities are far from obvious, as M. Itoh calculated. I think, however, other approaches would be hopeful, which idea came to me in 2000. I hope I could restart to pursue this train of thought.

I have other unpublished results on invariant theory, so that I should complete these studies.

Another theme is to investigate the works by Lambert, who lived about 300 years ago. There are lots of interesting things to look at. Among them, his W function is a particular special function, which has many different aspects with applications. Also, Lambert is known as quite a scholar, so that study on him will be fruitful even to the modern time.

What I have proposed are, though scatterd in some sense, are possibly unified in a background. It is my experience.

I once read through very old paper by Euler on the pentagonal number theorem, and got a new prospect from it. Also, from a paper by Turnbull, I got a new type of Capelli identities. So, from the Lambert work, I hopefully get something new to us.

Lambert's work on W function are, deeply related to solution of algebraic equations, apart from the pure "algebraic" approach. From this point of view, a new aspect for what Lagange, Euler, and Galois thought on the algebraic equations, would be obtained, I hope. It is very interesting point of view.

Another topic that I am interested in is the notion of "pseudo-topology". This notion is, on one hand a sort of anscetor of "topology", and actually, despite its name, a finer structure than topology on the other hand. Also it is also a natural structure from the categorical point of view. This notion has been noticed from the notion of 'topos' initiated by A. Grothendieck. It would be hopeful to re-consider the pseudo-topological structure in the framework of algebraic treatment of functional analysis.

Last year, I found a new proof for the Normal Basis Theorem on Galois extension, which comes as a corollary of duality argument of the Galois' fundamental theorem. Besides, a thorough investigation of the first paper by Galois himself would show a new point of view on the history of algebraic equation, as I mentioned above. Hopefully the study on Galois theory from the duality point of view will take us to a higher standpoint unifying several works. I would like to pursue this train of thought in near future.