Research plans

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I am going to devote myself to the following two projects as I explain below.

(1) Building a theory of representations of groupoids and its applications;

I wrote a paper [11] as a first step toward a reconstruction of the theory of generalized homology theories with which Hopf algebroids are associated as representation theory of groupoids by using the notion of fibered category. However, in this paper, I could only give right formulation of the notion of the representation of groupoids and the definitions of related basic notions such as "pull-back of representations by a morphism of groupoids", "regular representation" and "trivial representation". After that, I gave sufficient conditions for the existence of "right regular representations", "left regular representations", "right induced representations" and "left induced representations" and also gave their constructions by studying a deep structure of fibered category. On the other hand, in the process of the study, it turned out that there are no easy generalizations of the notions of "fixed points" and "obits", which are familiar in the theory of representations of groups, in the theory of representations of groupoids. This fact shows that very careful considerations are needed to give right notions which correspond to the notions of "fixed points" and "obits" in the theory of representations of groupoids. Hence I am still on the halfway to build a theory of representations of groupoids and I am going to keep considering the theory of representations of groupoids as it should be by studying concrete examples of representation of groupoids and related problems as I mention below.

- Establish a general framework for the cohomology of representations of groupoids by considering the notion of regular representations and develop the cohomology theory. For example, for a morphism $\boldsymbol{f}$ of groupoids, examine relations between the cohomology of the representations of groupoids and the cohomology of the induced representations of groupoids by $\boldsymbol{f}$.
- By introducing a notion of unipotent groupoid, generalize "Landweber's filtration theorem", which shows the existence of a good filtration on a comodule over the Hopf algebroid assciated with the complex cobordism theory, to a theorem of the representation of general groupoids.
- I defined "category of plots" which generalizes the category of diffeological spaces by using a Grothendieck site $(\mathcal{C}, J)$ and a functor F from $\mathcal{C}$ to the category of sets in [16]. This category is denoted by $\mathcal{P}_F(\mathcal{C}, J)$ below. Since the notion of a "fibration" in $\mathcal{P}_F(\mathcal{C}, J)$ is nothing but a representation of a groupoid in $\mathcal{P}_F(\mathcal{C}, J)$, $\mathcal{P}_F(\mathcal{C}, J)$ is one of categories in which we can develop a theory of representation of groupoids. It can be expected that a study on fibrations in $\mathcal{P}_F(\mathcal{C}, J)$ give keys to build a theory of representations of groupoids.

(2) Reconstruction of the theory of unstable modules over the Steenrod algebra from a viewpoint of representation theory;

In [10], I showed nine properties of the filtration on the Steenrod algebra defined from the "excess" of monomials of the Steenrod operators. We call a graded cocommutative Hopf algebra with a filtration which has first five properties out of these nine properties a generalized Steenrod algebra. Then, the notion of unstable modules over the Steenrod algebra is generalized to the notion of unstable modules over generalized Steenrod algebras. On the other hand, for a graded cocommutative Hopf algebra A^* of finite type over a field K, we denote by A_* the dual Hopf algebra of A^*. For a left A^*-module structure $\alpha : A^* \otimes_K V^* \to V^*$ of a graded finite dimensional vector space V^* over K, a right A_*-comodule structure $\alpha' : V^* \to V^* \otimes_K A_*$ called "Milnor coaction" is defined, hence V^* is regarded as a representation of the affine group scheme represented by A_*, which we denote by G_{A_*} below. If A^* is a generalized Steenrod algebra, we can define the notion of unstable representations of G_{A_*} as a notion which is equivalent to the notion of unstable module over A^*.

A theory of unstable modules over the Steenrod was developed by J. Lannes and others and the purpose of this project is to enrich their theory by dealing not only the Steenrod algebra but also generalized Steenrod algebras which are obtained as subalgebras or quotient algebras of the Steenrod algebra for example. Specifically, by showing the existence of a left adjoint of the tensor product of an unstable representation, I try to generalize the notion of Lannes' T-functor, study the relationship between the T-functor and induced representations and classify unstable injective modules.