

Previous research

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Trigonometric functions are functions with a period of 2π . Elliptic functions are complex functions of one variable with two periods and can be seen as a natural generalization of trigonometric functions. Abelian functions are complex multivariate functions with two or more periods and are a generalization of elliptic functions. Trigonometric and elliptic functions contributed to the development of mathematics and science. Since the 1990s, research in integrable systems and cryptography has required sophisticated knowledge of general abelian functions. My research aims to extend the sophisticated theory of trigonometric functions and elliptic functions to general abelian functions.

The Riemann theta function associated with an algebraic curve depends on the canonical homology basis of the algebraic curve. However, for general algebraic curves, constructing a specific canonical homology basis is a difficult problem. Given an algebraic curve defined using a specific algebraic equation, the problem of how to construct a function (a sigma function) that is algebraically determined only from the coefficients of the defining equation of the algebraic curve, without depending on a canonical homology basis, by adjusting the theta function, was posed by F. Klein in the 19th century. The case of a hyperelliptic curve with one point at infinity was solved by Klein. The case of plane curves called (n, s) curves, which contain hyperelliptic curves with one point at infinity, was solved by Buchstaber, Enolskii, Leykin and Nakayashiki. I solved this problem for telescopic curves containing (n, s) curves. I solved this problem for hyperelliptic curves with two points at infinity (joint research with Buchstaber). Since the sigma function is algebraically determined only from the coefficients of the defining equation of the algebraic curve, if we can construct a solution to a differential equation using the sigma function, it will be easier to investigate the behavior of the solution when the coefficients of the defining equation of the algebraic curve are changed to degenerate the curve. Onishi showed that the coefficients of the power series expansion of the sigma function associated with an (n, s) curve are polynomials with integer coefficients in the coefficients of the defining equation of the curve (Hurwitz integrality). I generalized the Hurwitz integrality of the sigma function to the case of telescopic curves. For elliptic curves and hyperelliptic curves of genus 2, sigma functions are defined over p -adic fields using the Hurwitz integrality, and are applied to number theory. We believe that our results will contribute to the further development of this research.

I considered the problem of describing abelian functions on algebraic curves of higher genus in terms of abelian functions on algebraic curves of lower genus (reduction of abelian functions). An algebraic curve for which there exists a degree 2 morphism to an elliptic curve is called bielliptic. I described abelian functions on bielliptic genus 2 hyperelliptic curves in terms of elliptic functions (joint research with Buchstaber). I described the abelian functions of bielliptic genus 3 hyperelliptic curves in terms of elliptic functions and abelian functions of genus 2 curves. We believe that our results will contribute to the visualization and numerical computation of abelian function solutions to differential equations.