Future research proposals

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Curvature flow has proven to be an extremely effective method for studying the geometric structure of Riemannian manifolds. Significant achievements have been made through various studies on curvature flow and its related geometric structures. The critical points of functionals play a crucial role in the study of singularities in curvature flow. In my future research, I plan to investigate the geometric structure of critical points of functionals and pursue the following two closely related research objectives:

1. Research on the classification of complete self-shrinkers, which are critical points of the Gauss area functional, and on complete λ-hypersurfaces, which are critical points of the area functional under Gauss volume-preserving variations.

Specifically:

(1) Extend Brendle's Alexandrov-type theorem for two-dimensional self-shrinkers to dimensions three and above.

(2) Generalize Brendle's proven Wiggly conjecture and Ilmanen's conjecture on planar domains in two directions. First, propose and study the Wiggly conjecture and Ilmanen conjecture for n-dimensional proper complete self-shrinkers. Next, formulate and investigate Wiggly-type and Ilmanen-type conjectures for proper complete λ-hypersurfaces, and examine whether counterexamples exist. Additionally, study the classification of complete λ-hypersurfaces.

2. Research on complete minimal hypersurfaces and complete hypersurfaces with constant mean curvature, which arise as critical points of the area functional and the area functional under volume-preserving variations.

As a generalization of the Bernstein theorem, do Carmo-Peng, Fischer-Colbrie-Schoen, and Pogorelov showed that an oriented, stable, complete minimal surface in R^3 must be a plane. However, for $n > 6$, there

exist oriented, non-planar, non-graph, stable complete minimal hypersurfaces in R^{n+1}, leading to the following well-known conjecture:

Conjecture: For $n < 7$, an oriented, stable, complete minimal hypersurface in R^{n+1} must be a hyperplane.

We will study this conjecture. Furthermore, we will also investigate δ-stable complete minimal hypersurfaces and stable complete hypersurfaces with constant mean curvature.