Future Research

Tatsuki EZAWA
e-mail: m14006q@math.nagoya-u.ac.jp

As a continuation of the research conducted in fiscal year 2025, the goal of this study is to obtain the following two developments of the result in [1]: (1) an extension to a theorem in CAT(1) spaces, and (2) a replacement of the algorithm by a projection method based on contractions.

In both directions (1) and (2), although the conjectures proposed in fiscal year 2025 are believed to be reasonably valid, the assumptions imposed were overly general. As an initial objective, we will introduce certain compromises, such as strengthening the assumptions, in order to first obtain the desired results.

More specifically, in (1), while it is ideal to impose assumptions on each mapping that still include a wide range of examples in CAT(1) spaces, we will first attempt to establish the results under assumptions such as nonexpansiveness. In particular, under the nonexpansive assumption, we have previously obtained nontrivial results, and subsequently refined the arguments in the proofs to derive results under more appropriate assumptions. Furthermore, in several previous studies on CAT(1) spaces, the nonexpansive assumption has been employed, and we believe that their techniques can be adapted to our setting.

In (2), as is also planned in (1), when dealing with multiple mappings, we will first complete a concrete proof for the case of only two mappings. This work is currently in progress and constitutes the most essential part of the study. Based on our past experience, once a result is obtained for two mappings, it is often straightforward to generalize it to finitely many mappings.

Moreover, naturally arising from (1) and (2), a further problem (3) emerges: replacing the algorithm in the preceding work [1] by a contraction-based projection method on CAT(1) spaces. For this problem, we will carefully examine whether the desired result can be obtained simply by combining the proofs of (1) and (2).

We believe that solving (1) and (2) will lead to publishable results. However, the extent to which the assumptions on each mapping can be refined to more general ones will be determined after studying additional literature to better understand concrete examples and the validity of such assumptions. The structure of the resulting paper will also be decided depending on how similar or independent the proofs are, including those for (3).

Finally, since problems of this nature have previously been obtained by modifying the underlying space, the assumptions on mappings, and the methods of handling multiple mappings, we will continue to pay close attention to both past and recent related literature. If interesting problems arise, they will be considered as research topics. To this end, we will actively collect information at workshops and conferences and incorporate opinions from other specialists. In this regard, there is also the possibility that this research will develop into a joint project with Dr. Yasuki Kimura (Toho University) and Dr. Keisuke Shindo (Hachinohe National College of Technology), with whom we have previously discussed these topics and received valuable advice.

Reference

[1] M. A. A. Khan and P. Cholamjiak, A multi-step approximant for fixed point problem and convex optimization problem in Hadamard spaces, Journal of Fixed Point Theory and Applications, Vol. 22, No. 3, 2020.