(2) Research plan

" Derivation of wave energy and nonlinear mean flow through Nambu mechanics and its application to the stability of a compressible shear layer"

Nambu bracket is a magic wand that lies at the core of non-canonical Hamiltonian formalisms. For 2020-22, I chaired the OCAMI symposiums *"Nambu mechanics for linking space-time topology with formation of micro-macro magneto-vortical structure"*, which enlightened space-time topology. Using the deepened Nambu mechanics[115)] as a lever, I will derive nonlinear quantities such as energy and wave-induced mean flow and tackle the puzzles that hang over a compressible shear layer.

1. Deepening non-canonical Hamiltonian structure for fluid mechanics and magneto-hydrodynamics (MHD) by Nambu brackets

Non-canonical Poisson brackets for fluid dynamics and MHD take complicated forms. Its structure becomes clear if we derive the Nambu-bracket representation by a hint that the Casimir invariants of MHD are exhausted by four, including the cross helicity. Moreover, Arnold's theorem which characterizes stationary solutions of incompressible Euler flows and gives the wave energy formula, can be extended to compressible MHD.

i) The Nambu bracket representation is sought for the extended MHD (Hall effect, electron inertial effect) and characterize the stationary solution.

ii) Along an extension of Arnold's theorem lies the wave energy and the wave-induced mean flow. The isomagnetovortical disturbance is constructed with an example for a compressible shear layer.

iii) The linearized MHD equation for the Lagrangian displacement is the second-order Friedman-Rotenberg (FR) equation. The wave energy is derived by factoring the FR equation.

iv) Furthermore, the mean flow induced by wave interactions is calculated by extending the isomagnetic circulation disturbances up to second order in amplitude.

2. Instability of a vortex sheet and a shear layer in a compressible fluid

After a half century since the Concorde, development of supersonic aircrafts has resumed, and vortex dynamics in the supersonic regime has attracted attention. In the incompressible approximation, a vortex sheet, which is the limit of a shear layer with a thickness of 0, always causes Kelvin-Helmholtz (KH) instability, amplifying two-dimensional wavy deformation, and the amplification rate is proportional to the velocity difference ΔU. Compressibility suppresses it when ΔU becomes even larger and the Mach number $M=\Delta U/c$ (c: sound speed) exceeds $\sqrt{8}$. This surprising result is limited to two dimensions, and larger ΔU is required to suppress three-dimensional instability. In addition, another instability resides inside the shear layer. Armed with **the wave energy formula for compressible fluids**, I will unravel the tangled threads in stability of a compressible shear layer.

i) I will clarify compressibility stabilization from the perspective of negative wave energy and over-reflection caused by negative momentum.

ii) Instability localized within the finite shear layer will be characterized using the Hamiltonian spectral theory. Calculation of the energy of waves standing on the shear layer holds the key.

iii) The effects of gravity and surface tension on compressible KH instability will be investigated. Preliminary calculations show that gravity destabilizes the interface even when a light fluid is above a heavy one. So is the surface tension. It is a puzzle that these restoring forces act to destabilize the interface. This puzzle is to be resolved by appealing to the energetics.