

(2a) Summary of research results

The world surrounding us is filled with vortical structures, from micro to macro scales, even the galactic scale. I am engaged in developing mathematical methods for dealing with nonlinear dynamics and stability of *vortices*, which are constituents of fluids. I am pursuing "*topological fluid dynamics*", which arises from the property that magnetic and vorticity fields are frozen into fluids.

1. Singular perturbation methods for motion of a vortex of finite thickness in a viscous fluid

- i) Exploiting the dipole structure inherent to finite core thickness, a third-order correction term, in the ratio of core to curvature radii, to the speed of a vortex ring was derived by applying a singular perturbation method to the Navier-Stokes equations²⁷.
- ii) For the two-dimensional high Reynolds number motion of a pair of counter-rotating vortices in an incompressible fluid, matched asymptotic expansions was extended up to fifth order in the ratio of the core radius to vortex distance⁹¹.
- iii) For magnetic vortex rings containing magnetic flux for MHD with the Hall effect, we gave a general velocity formula for thin rings and an exact solution for spherical vortices for fat ones¹¹⁹.

2. 3D stability theory of vortex tubes

I made 3D stability analyses of vortex tubes based on Krein's theory of Hamiltonian spectra.

- i) The growth rates and eigenfunctions were written out for the 3D instability of a vortex tube embedded in a straining field³⁵.
- ii) The "*curvature instability*" was found for Kelvin's vortex ring, a resonance between two Kelvin waves with azimuthal-wavenumber difference 1, caused by the curvature effect^{34,38}.

3. Flow spectrum and nonlinear interaction of waves

By extending Arnold's theorem for incompressible Euler flows to compressible magneto-hydrodynamics (MHD), we constructed the isomagnetovortical perturbations and derived a new formula for wave energy^{50,51,112}.

4. Characterization of topological invariants by Noether's theorem and Nambu mechanics

- i) By Noether's theorem in the framework of Hamilton's principle of least action for MHD, the "*cross helicity*" is characterized as integral for the particle relabeling symmetry^{52,71,115}.
- ii) In MHD, the Hamiltonian equations of motion were expressed by Nambu brackets with use of three Casimir invariants, *cross helicity*, the *total entropy*, and the *magnetic helicity*¹¹⁵.
- iii) The cross-helicity of plasma and MHD has been extended to special relativity.

5. Effect of gravity waves on the Kelvin-Helmholtz instability of a shallow-water flow

A surface of tangential velocity discontinuities is susceptible to the Kelvin-Helmholtz instability (KHI). In shallow water flows, KHI is stabilized when the velocity jump exceeds 2.82 times the propagation speed of the gravity wave. The effect of finite width of shear layer was calculated¹¹⁴.

6. Creation of chemical materials using materials informatics

An algorithm of combining quantum chemical simulations with machine learning is constructed to generate ring compounds with desired properties. Its program is made and executed^{109,113}.