

Preceding results

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I concern functional inequalities such as Hardy-type or uncertainty principle inequalities for constrained vector fields. The Hardy inequality, first found by G. H. Hardy for one-dimensional scalar fields, was developed by J. Leray to that for higher-dimensional vector fields, in the context of Navier-Stokes equations. Today it is applied to many directions. A matter of special interest for me is how the best constants in the functional inequalities change from the original ones when the test functions are vector-valued and are subject to differential constraint such as curl-free or solenoidal condition. This problem originates from the one suggested by Costin-Maz'ya in 2007, who derived a new best constant in the Hardy-Leray inequality for axisymmetric solenoidal fields. In view of their results, I tried to remove the axisymmetry condition or extend such results to other types of inequalities. On this situation, I have ever obtained the following results:

Sharp Hardy-Leray inequality for solenoidal fields

For the case of three-dimensional solenoidal fields, I found in a joint work [9] with Prof. F. Takahashi that the same best constant as Costin-Maz'ya can be obtained by restricting the axisymmetry condition on only the azimuthal components of the test fields. Furthermore, in [2] and [7] I extended this result to the case of general-dimensional solenoidal fields without any symmetry assumption at all, and incidentally found in [4] a simpler expression for the best constant in view of the dependence on the weight exponent.

Sharp Rellich-Leray inequality for solenoidal fields

As a second-order version of Hardy inequality, I derived in the paper [1] the new best constants in Rellich inequality with weights for axisymmetric solenoidal fields, by giving a rigorous characterization of axisymmetric vector fields. Moreover, this result was extended in [3] to the case of general solenoidal fields, and the non-attainability of the best constants was also shown.

Sharp Hardy-Leray and Rellich-Leray inequalities for curl-free fields

Following from the two-dimensional case of Costin-Maz'ya's result, in a joint work [11] with Prof. F. Takahashi, I derived sharp constants in weighted Hardy and Rellich inequalities for higher-dimensional curl-free vector fields. We also proved in [12] the non-attainability of the best constants found in the former work, by estimating remainder terms. With regard to the critical case, we verified in [10] that the best constant in the logarithmic weighted Hardy inequality for two-dimensional curl-free fields coincides with the one for unconstrained fields.

Sharp Rellich-Hardy inequality for curl-free or solenoidal fields

With regard to the Rellich-Hardy inequality found by Tertikas-Zographopoulos, which can be considered as an intermediate between Hardy and Rellich inequalities, the best constant for curl-free or solenoidal fields was computed in recent papers [13] and [8] including any radial power weight.

Sharp uncertainty principle inequality for solenoidal fields

Heisenberg's uncertainty principle inequality is well known for scalar fields together with the explicit best constant. In [5, 6] I derived the new best constant in the same inequality for solenoidal vector fields and also found that it can be attained by a class of solenoidal fields. Moreover, the profiles of the extremal functions was clarified.