

Research Plan (Shota Hamanaka)

- Synthetic scalar curvature bounds and the Ricci flow : I'm interested in “weak notions of (total) scalar curvature lower bounds” and “appropriate topology to handling the space of all space with a (total) scalar curvature lower bound in a certain weak sense”. These relate to some almost rigidity theorems for the (total) scalar curvature. Thus, I'd like to investigate the relations between some weak notions of (total) scalar curvature lower bounds (including the notion defined in my previous paper [9]). I also want to investigate those effectiveness of dealing with some almost rigidity problems for the (total) scalar curvature.

Sormani–Tian–Wang [17] constructed a concrete example of space with non-negative scalar curvature in a certain weak sense. In particular, I want to check whether their example meets the another definition by Burkhart–Guim [3]. And, relating to [10], I also want to investigate the weak notions of the **upper** bound of the (total) scalar curvature. As far as I know, there is currently no research about this.

- Analysis of the Ricci flow on a closed four-dimensional manifold: Gauge theoretical approaches have been widely successful in the topology of four-dimensional manifolds. On the other hand, Bamler [2] has suggested the effectiveness of geometric analytical methods using Ricci flow. I want to approach four-dimensional topology in this geometric analytic direction. To do so, I want to study the behavior of the first singular time of the Ricci flow with uniformly bounded scalar curvature on a closed four manifold.
- Geometric analysis on weighted Riemannian manifolds: An weighted Riemannian manifold (or smooth metric measure space) is a pair of Riemannian manifold and smooth measure on it. Here, the equipped measure is written in terms of a function and the Riemannian volume measure. Such spaces are known to appear in Ricci flow theory, Ricci limit spaces, and also in certain contexts of High energy physics. Especially, Baldauf–Ozuch [1] recently defined the weighted ADM mass for the weighted asymptotically flat space, and proved a certain type of positive mass theorem for spin weighted asymptotically flat spaces. Furthermore, Chu-Zhu [7] proved a certain type of positive mass theorem for (not necessary spin) weighted asymptotically flat space of dimension between three and seven¹. On the other hand, there are some studies about the behavior of the unweighted ADM mass under some geometric flows ([8], [15], [6], [5]). Taking these things account, we are investigating in [12] the behavior of the weighted ADM mass along the weighted Yamabe flow. In our future work, we will study about an alternative proof of the weighted positive mass theorem using some geometric flows and the lower semicontinuity of the weighted ADM mass under some (weighted) convergence of weighted asymptotically flat spaces.
- Extremal metrics involving scalar curvature : Listing [16] proved certain types of rigidity theorems involving the quantity $R_g \cdot g$ (Scalar curvature \times Metric). On the other hand, this quantity appears in the sufficient condition for metric to be a positive Yamabe metric in its conformal class [13]. Taking this account, I want to investigate the relation between Yamabe metrics or Einstein metrics and metrics for which Listing's rigidity theorems hold. In [11], I proved that for a metric g_0 on a closed n -manifold M^n if the norm of the traceless Ricci tensor of g_0 is not vanishing (hence g_0 is not Einstein) and the scalar curvature and the Ricci curvature satisfies a certain condition, then Listing's rigidity theorems do not hold for g_0 ².

¹More recently, Law–Lopez–Santiago proved that in both cases (i.e., spin and non-spin cases), stronger forms of the statements actually hold, and they are equivalent to the original unweighted positive mass theorem.

²I also gave some examples of Riemannian manifolds satisfying this assumption.

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