- Around Yamabe problems : In [1], I extended Koiso's decomposition theorem on closed manifolds to compact manifolds with boundary. As an application, I also give some uniqueness theorems of relative Yamabe metric.

  In [2], I gave a sufficient condition for a constant-scalar-curvature metric with minimal boundary to be a relative Yamabe metric on a compact manifold with boundary. Using this condition, I also gave some examples of non-Einstein positive relative Yamabe metrics. In [9], we gave a sufficient condition for a scalar-flat metric with constant-mean-curvature boundary to be a minimizer of the corresponding Einstein–Hilbert functional. In the same paper, we also gave some examples of Riemannian manifolds for which Obata's theorem and its opposite does not hold. In [8], I investigated some relations between Yamabe metrics and Riemannian metrics for which certain rigidity theorems by Listing does not hold.
- Geometric flows : In [3], I proved that n-simensional closed Ricci flow $g(t)\,(t \in [0,T), T < \infty)$ with a uniformly bounded certain integral energy of the scalar curvature $(n = 4)$ or the Riemannian curvature $(n \geq 5)$ converges to a smooth Riemannian manifold away from finitely many orbifold singularities as $t \to T$, in the sense of Cheeger–Gromov sense. In [4], I investigated the behavior of a certain type of the Ricci flow with uniformly bounded scalar curvature on an n-dimensional closed manifold at the first singular time. In [10], we investigated the convergence rate of the weighted conformal mean curvature flows on smooth metric measure spaces.
- Generalizations of (total) scalar curvature bounds : In [5], I proved some Gromov-types of limit theorems for the total scalar curvature. I used a certain type of rigidity of the Ricci flow to prove those. At least for experts, it was already known that the (unweighted) total scalar curvature behaves continuously under the $C^0 \cap W^{1,2}$-convergence of metrics. On the other hand, in this paper, I proved that the lower bound of a certain weighted total scalar curvature is preserved under a certain weak notion of convergence of metrics and potential functions. In particular, this implies that the lower bound of Perealman's $\mathcal{F}$ functional and $\mathcal{W}$ functional are preserved under a certain weak notion of convergence of metrics and potenital functions. Furthermore, as an application, I also gave a definition of scalar curvature lower bound in a weak sense. In contrast, I proved in [6] that for a fixed conformal class C on a closed manifold, the **upper** bound of the total scalar curvature is preserved under the C^0-convergence of metrics in C^1. I used a certain type of rigidity of the Yamabe flow to prove that.
- Codimension one submanifolds and Geometry of manifolds with scalar curvature lower bound : In [7], I investigated that the decay rate of the scalar curvature of a complete non-compact nonparabolic steady gradient Ricci soliton in terms of the distance from a fixed point. In order to prove that, I used μ-bubble method which is a generalization of Schoen–Yau's minimal hypersurface method. Moreover, using a similar method, I also gave an alternative proof of a certain Myers' type of theorem for shrinking gradient Ricci solitons.

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