Research Plan

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I plan to study rational structures of equivariant twisted D-modules.

If a Harish-Chandra pair $(\mathfrak{g}, K)$ over the field $\mathbb{C}$ of complex numbers or an algebraic closure $\bar{\mathbb{Q}}$ of the field of rational numbers admits an F-form for a subfield F, it is natural to ask **existence of F-forms of irreducible $(\mathfrak{g}, K)$-modules and classification of irreducible modules over F**. In particular, these problems for cohomologically induced modules are related to number theory. A possible approach to them is to consider equivariant twisted D-modules. In fact, there is a **categorical equivalence between the category of $(\mathfrak{g}, K)$-modules (with regular infinitesimal character) and the category of equivariant quasi-coherent twisted D-modules on the flag variety** (the Beilinson–Bernstein equivalence). This is known over algebraically closed fields of characteristic zero. This equivalence holds true without the algebraically closed assumption under an appropriate formulation (Preprint 2).

I next mention an approach to rational structures. Loewy gave **a classification scheme of finite dimensional real irreducible representations of a group G** and determined **the division algebras of their endomorphisms**. According to him, there is a natural bijection between the set of isomorphism classes of finite dimensional real irreducible representations of G and that of complex conjugacy classes of isomorphism classes of finite dimensional complex irreducible representations of G. He also showed that for a finite dimensional (self-conjugate) complex irreducible representation of G, the division algebra of endomorphisms of the real irreducible representation corresponding to V is determined by the so-called **index**. It is a sign **determined by V**.

Then Tits gave a similar classification result for connected reductive algebraic groups G over a field F of characteristic zero. That is, he gave a bijection between the set of isomorphism classes of irreducible representations of G and that of Galois conjugacy classes of isomorphism classes of irreducible representations of $\bar{F} \otimes_F G$, where $\bar{F}$ is an algebraic closure of F. For a self-conjugate irreducible representation V of $\bar{F} \otimes_F G$, he gave a **2-cocycle which determine (the opposite algebra to) the division algebra of endomorphisms of the irreducible representation of G corresponding to V**. To be precise, this cocycle was introduced by Borel–Tits as an obstruction class to existence of an F-form of V. We call it the **Borel–Tits cocycle** of V. This is determined by V. For general V, we can reduce ourselves to the self-conjugate case by taking the base change of G to the field of rationality of V. Note that this result is **valid for general affine group schemes**. Then we can regard it as a generalization of Loewy's result explained above (for G finite). The **Borel–Tits cocycles generalize the indices**.

In Paper 1, I established **categorical frameworks in which the results of Loewy, Borel–Tits, and Tits remain valid. I will show that these frameworks apply to equivariant coherent twisted D-modules**. In particular, I will formulate the problem concretely in the study of the rationality of equivariant twisted D-modules: namely, I will show that **the essential objects to be analyzed are the Galois actions and the associated Borel–Tits cocycles**. I will also compute these explicitly in the case of flag varieties.