

(英文)

This researcher has conducted qualitative and quantitative research on dynamical systems. They are described below.

**(Qualitative Research)** First, he focused on the following two properties of hyperbolic dynamical systems: (A) expansivity, and (B) pseudo orbit tracing property. (A) implies a sensitive dependence of the dynamical system on initial values, and (B) implies that a computer simulation computes the true trajectory for some initial value. A homeomorphism on a compact metric space with these two properties has a Markov partition. Moreover, it was proved that the two-dimensional homeomorphism is topologically conjugate to the algebraic example on the torus, as well as Anosov diffeomorphisms. He also showed that it is topologically conjugate to the algebraic example on the torus in higher dimensions. In the proof, he introduced the concept of generalized foliations, computed the fixed point index, and counted the number of periodic points. Then, by using Lefschetz's fixed point theorem, the homotopy class was determined, and a conjugate map was constructed. For a surface homeomorphism with the property (A) in dimension 2, he constructed two transverse invariant measured singular foliations. This says that the homeomorphism with (A) is identical to the pseudo-Anosov map due to Thurston. In addition, he negatively solved the problem that had been considered difficult to solve since the 1950s; "Does an expansive homeomorphism exist on the two-dimensional sphere?". As a result, he was asked to give a special lecture and write a paper for the Mathematical Society of Japan. In addition to the above, he proved a number of new results for discrete dynamical systems with properties (A) and (B), and they were published together as a book in 1994. The book has been highly acclaimed, has been book-reviewed in Bulletin of the AMS, etc., and continues to be cited in many papers. These were the results of his research until around 2000. He was asked to write a new entry "Hyperbolic dynamical systems" in the 4th edition of the Iwanami Dictionary of Mathematics, probably because of the results of his research. He is continuing his research to solve the difficult problems raised by Smale and others in the 1960s.

**(Quantitative Research)** In addition to the above research, he started an experimental mathematical research using a computer around 2000 in collaboration with Chihiro Matsuoka as a quantitative study of dynamical systems. They used the Borel-Laplace transformation, called resurgent analysis, developed by Ecalle and others, to obtain asymptotic expansion functions describing stable (unstable) manifolds of dynamical systems defined by polynomials such as the Henon map. They have succeeded in calculating topological entropy and Lyapunov exponents in concrete terms. Very recently, they discovered a quite simple method that allows us to obtain the same function without using the resurgent analysis. The new method is extremely versatile and can be applied to dynamical systems with analytic functions more general than polynomials, including higher-dimensional dynamical systems. They are continuing this research to elucidate many phenomena.