Research plans
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Reference numbers correspond to the attached list of research achievements.

My future research will focus on mathematical models involving parabolic-type equations. By extracting the intrinsic energy-dissipation and variational structures of these systems, I aim to establish a unified understanding of qualitative properties and long-time behavior of solutions. Chemotaxis systems serve as a central theme. Their nonlocal nonlinear interaction terms and fully parabolic structures often invalidate standard analytical methods, motivating the development of new techniques. The main research directions are as follows.

I. Critical structures and global dynamics in chemotaxis with nonlinear diffusion

I will study chemotaxis equations with nonlinear diffusion $\partial_t u - \nabla \cdot (u^{\alpha-1}\nabla u)$ ($\alpha \in \mathbb{R}$), where the critical exponent $\alpha_{c,n} = 2 - 4/n$ determines the transition in global dynamics. In joint work with Ph. Laurençot, critical phenomena have been clarified for linear diffusion in $n = 4$ and degenerate diffusion in $n \geq 5$. For lower dimensions $n \leq 3$, the interplay between fast diffusion and nonlocal chemotactic effects produces unexpected behaviors, and existing methods often break down. I will analyze how diffusion and concentration compete near the critical regime and identify fundamental mechanisms governing long-time dynamics.

II. Mass-critical behavior in the fully parabolic Keller–Segel system in 2D

For the 2D fully parabolic Keller–Segel system, the mass threshold $M = 8\pi$ separates global existence from singularity formation. This project aims to establish infinite-time blow-up at the critical mass and study stability of steady states. Unlike the parabolic–elliptic case, the fully parabolic system lacks conservation of the second moment and center of mass, and finite-time blow-up techniques cannot be directly applied to infinite-time singularities. I will develop energy-dissipation estimates for radially symmetric solutions, ODE-type energy analysis, and local analysis. As an initial step, I plan to prove uniform boundedness of global solutions for $M < 8\pi$.

III. Chemotaxis with logarithmic sensitivity

I will investigate chemotaxis equations with logarithmic sensitivity, for which the global behavior depends on the perceptive coefficient χ, and the threshold $\chi^* = n/(n-2)_+$ is conjectured to be critical. While the 2D case has been resolved by Fujie–Senba (2016), higher-dimensional cases remain open. The steady-state problem is closely related to semilinear elliptic equations appearing in degenerate Keller–Segel systems. By exploring these connections and underlying variational structures, I will work toward clarifying the critical phenomena associated with logarithmic sensitivity.

IV. Other International collaboration

I am continuing collaborative work with T. Cieślak on thermoelastic systems, which equip with mechanical energy conservation with entropy dissipation. We aim to describe asymptotic behavior under minimal regularity assumptions. During my current research stay in France, I am also collaborating with Ariane Trescases (CNRS & Université Toulouse) and Filippo Santambrogio (Université Lyon 1), and plan to further develop these partnerships.