Research Summary
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Reference numbers correspond to the attached list of research achievements.

■ Research Summary

My research focuses on nonlinear partial differential equations, in particular chemotaxis (Keller–Segel) systems describing cell movement. These systems combine diffusion and nonlinear advection, producing competing effects of spreading and aggregation. Due to mass conservation, the balance between these effects leads to mass-critical thresholds that determine global stability and blow-up behavior. In two dimensions, for instance, blow-up occurs when the initial mass exceeds 8π. My work has analyzed such critical structures using conserved quantities, energy methods, and variational frameworks.

▶ Global Dynamics in Higher Dimensions (Supercritical Regimes) ([1, 5])

In dimensions $n \geq 3$, the system becomes L^1-supercritical. Using Shannon-type inequalities with optimal constants, I derived criteria for initial data that induce finite-time blow-up and established smallness conditions ensuring global existence. These results clarify the threshold between stable and unstable behaviors in supercritical settings.

▶ Mass Thresholds and Global Behavior in Higher Dimensions ([2, 3, 4, 7])

Results [2, 3]: For a simplified parabolic–elliptic system in four dimensions, I identified the mass-critical threshold as $(8\pi)^2$. Using Brézis–Merle type inequalities together with optimized rearrangement-based inequalities, I proved global existence for initial masses below this threshold.

Result [4]: For fully parabolic chemotaxis systems, optimal regularity is difficult to ensure. By exploiting a variational structure that relates the parabolic system to its elliptic counterpart, I established global-in-time solutions. To overcome difficulties in controlling far-field behavior, I introduced a new energy functional that provides uniform bounds for solutions under appropriate mass conditions.

Result [7]: For parabolic–elliptic systems on $\mathbb{R}^n$, I determined dimension-dependent mass-critical thresholds using a monotonicity formula and analyzed how the shape of initial data affects long-time solution behavior.

▶ Fisher Information and Quasilinear Keller–Segel Systems ([6])

I first studied generalized nonlinear diffusion equations of the form $\partial_t u = \nabla\cdot(a(u)\nabla u)$ and established monotonic decay of a Fisher-information-type energy functional, together with related functional inequalities. In one dimension, I also derived an additional monotonicity property specific to the 1D setting.

As an application, I analyzed a quasilinear fully parabolic Keller–Segel system with nonlinear diffusion and sensitivity. In the one-dimensional L^1-critical regime, I proved the existence of global solutions for all initial masses, demonstrating that singularities never occur—contrasting with the 2D and higher-dimensional cases. The Fisher-information-based functional enabled stronger regularity estimates than those obtainable through classical entropy methods alone.