(2) Research Plan

Hiroki Imai

Background of the research

Higher dimensional gauge theory with orbifolds as an extra dimension have been studied as one of candidates to go beyond the standard model in particle physics. In general, choice of boundary conditions of a field on the orbifold enriches low energy effective theories realized in lower dimensional spacetime. For example, it is known that a chiral fermion on a six-dimensional spacetime with compactified $T^2/\mathbb{Z}_2$ with $U(1)$ background gauge field has 16 patterns determined by the data of boundary condition [1].

Considering a non-abelian gauge theory on a higher dimensional spacetime with an orbifold as an extra dimensions, its four-dimensional effective field theories are different depending on choice of the data of boundary condition in general. However, if two boundary condition data seem to be different, they are known to be possible equivalence up to large gauge transformation. It had not completed comprehensive classification of the equivalence classes for a long time, but recent years it has done about $SU(N)$ and $SO(N)$ gauge theory with $T^2/\mathbb{Z}_m\,(m = 2, 3, 4, 6)$ as compactified space [2]. A trace of gauge group action for a matter field on an orbifold fixed point is an invariant, thus we can perform systematic classification without conventional heuristic approach.

On the other hand, the previous researches assumed the flat gauge connection (i.e. field strength vanishes). If the higher dimensional gauge theory has 't Hooft flux, it was pointed out that there appears a new class of rank reducing of gauge group [3]. Orbifolding also causes nontrivial rank reducing, thus we expect that combining the both enriched the reducing patterns. It is a worthwhile task to classify the rank reducing patterns. Also, it is nontrivial that the trace on an orbifold fixed point is invariant.

Purpose of the research

Complete the classification of $T^2/\mathbb{Z}_m$ orbifold boundary condition data in gauge theories with 't Hooft flux.

Significance of the research

The aim of the research is to provide a useful framework for future model buildings to go beyond the standard model in particle physics or searching string landscape. It remains unclear whether the conventional classification performed by heuristic method has completed enough. Against this background, the research plans to perform recently developed systematic approach [2] to cover physically possible classes. This problem is mathematically known as classification of moduli space on an orbifold up to gauge equivalence, thus it is expected to bring new perspective it introducing 't Hooft flux provides nontrivial results.

Research contents

In this research, we focus on gauge theories with $\mathfrak{su}(N)$ as a Lie algebra. We start 6d Yang-Mills theory on $\mathbb{R}^{1,3} \times T^2/\mathbb{Z}_n\,(n = 2, 3, 4, 6)$ with the $\mathbb{Z}_m$-valued 't Hooft twisted boundary condition. We will complete classification of boundary condition data up to gauge equivalence. 't Hooft twist and orbifoldings are known methods to reduce rank of gauge group [3], thus we classify patterns how to break the gauge group when both coexist.

Research methods

Plan1: First, we check whether the systematic approach is valid for this case. We test the case which the 't Hooft flux does not localize at orbifold fixed points. We split the boundary condition data as 't Hooft twist part and non-abelian Wilson line part [3]. Taking $K = \gcd(N, m)$, the Wilson line part is described by $SU(K)$ gauge group. Based on the prescription given by literature [2], we classify the boundary condition data by taking trace on the fixed points.

Plan2: Next, we consider general case within 't Hooft flux localization on orbifold fixed points. We try to classify gauge equivalence class taking trace on the orbifold fixed points. We will classify with different localization flux on each fixed point, the boundary condition data is not necessarily split. We will perform the research with confirming the result obtained in the Plan1.

[1] T-h. Abe, et. al., JHEP01 2014 (2014) 065

[2] K. Takeuchi, T. Inagaki, PTEP 2024 (2024) 3, 033B03, PTEP 2024 (2024) 6, 063B04, PTEP 2025 (2025) 4, 043B03,

[3] G. von Gersdorff, Nucl. Phys. B 793 (2008) 192–210