(2) Summary of research history

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Higher dimensional theories with a compactified spatial dimension (so-called extra dimension) has been studied to reveal structures of quarks and leptons from hidden geometry. For example, the fact that quarks and leptons has three copies is called the generation problem and background magnetic flux on the extra dimension is considered as a candidate of model to describe it. The magnetic flux is quantized due to periodicity of the extra dimension and the quantized number is combined the number of chiral zero modes through the index theorem. Against this mathematical background, a scenario has been considered that the generation number is identified with the number of chiral zero modes on the extra dimension and is topologically protected thus robust. It is also known that a simple setup constructs a complicated structure if we treat the extra dimension as an orbifold. A lot of researches about orbifolds have been done as a candidate to describe parameter structure in terms of fermion mass structures.

An orbifold is constructed from a manifold imposed an identification under a finite group action. In particular, considering a plane with double periodicity (i.e. a two-dimensional torus) T^2 and dividing it by discrete rotational group $\mathbb{Z}_N$, we obtain $\mathbb{Z}_N$ $(N = 2, 3, 4, 6)$ with some isolated conical singularities. For example, the case of $N = 2$ can be depicted a schematic diagram of a pillowcase illustrated in Fig. 1.

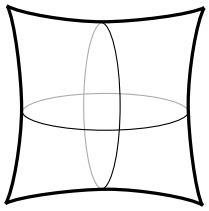

Figure 1: a schematic illustration of an orbifold $T^2/\mathbb{Z}_2$. We can see four conical singularities as corners.

At first, we give a proof of the index theorem on $T^2/\mathbb{Z}_2$ with uniform magnetic flux, by constructing mode functions and computing the index based on Fujikawa's chiral anomaly computation [1]. We also provided derivation for the case of $T^2/\mathbb{Z}_N$ $(N = 3, 4, 6)$ under an assumption. We clarified that the index theorem combine chiral zero modes with topological invariant on the extra dimension and the index is described by winding numbers at the orbifold fixed points in addition to the magnetic flux conventionally considered.

Next, we complete classification of patterns compatible with Yukawa couplings on $T^2/\mathbb{Z}_2$ and clarified the topological structure [2]. The previous paper [1] showed that patterns with the same number of chiral zero modes are classified by the winding number at the fixed points. Thus, we considered a set up with a left-handed fermion and a right-handed one and examine compatibility to the three working assumptions to be healthy as a higher dimensional model:

(1) The number of left- and the right-handed chiral zero modes is same.
(2) We can perturbatively introduce a 6d Yukawa interaction between both chiral fermions.
(3) There can be exist an intermediate field between the fermions via the Yukawa interaction.

As a result of these requirements, we found that it does not satisfy (1)-(3) simultaneously the case that there coexist a fixed point where only the left-handed fermion has nonzero winding number and one where the right-handed fermion has so (See Fig. 2). This case is incompatible with high energy model. Against conventional high energy model buildings, we required minimal inputs as phenomenological data and succeeded to obtain a result. For example, we were not required information as the three-generation.

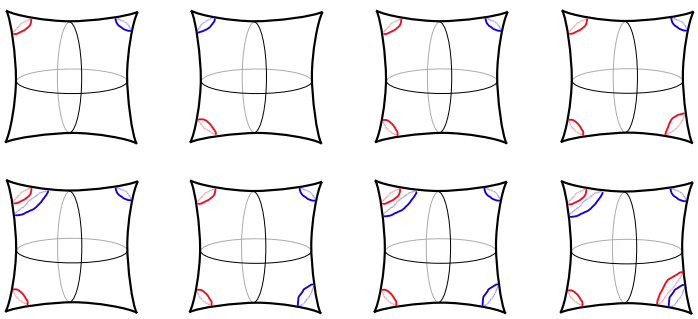

Figure 2: The incompatible $T^2/\mathbb{Z}_2$ model. red : winding of the left-handed fermion blue : winding of the right-handed fermion