Future Research Plans

Hiroshi Ishimoto

Newforms of half-integral weight and local new vectors of the metaplectic group Mp(4)

I will investigate newforms as a development of Ibukiyama's isomorphism. There is an information called the level of modular forms. A modular form of smaller level is called a newform. In the case of degree 1, there is a study of newforms in the results of Shimura, given by Ueda--Yamana. Therefore, following this, I will study newforms in Ibukiyama's isomorphism in the context of modular forms of half-integral weight and of degree 2. On the other hand, a local new vector is a vector in a representation of a group over a local field, which is a local component of an automorphic representation, corresponding to a newform. In this project, we investigate the existence and the number of local new vectors in the representations of Mp(4) and newforms of half-integral weight and of degree 2.

First of all, for newform, the case of integral weight is well known, so we get results by constructing a similar isomorphism between them of half-integral weight and integral weight as Ibukiyama's isomorphism. There is a prior study by Ueda--Yamana in the case of degree 1. Referring to this, I will make predictions, and prove it by applying the method of the proof ([Is2]) of Ibukiyama's isomorphism. On the other hand, for local new vectors, I will extend a previous study in the case of the degree 1 by Roberts--Schmidt to the case of degree 2. Roberts--Schmidt's study uses Waldspurger's results to investigate the existence and the number of local new vectors of Mp(2). I will utilize the results of Gan-Savin and Gan--Ichino, which are generalizations of the studies of Waldspurger. Finally, I will compare the results of researches of newforms and of local new vectors.

Ibukiyama's isomorphism and Theta correspondence

Ibukiyama's isomorphism is an isomorphism

$$S^{+}_{k-\frac{1}{2},j}(\Gamma_0(4),\psi) \cong S_{j+3,2k-6}\big(\mathrm{Sp}(4,\mathbb{Z})\big),$$

of vector spaces which preserves L-functions, for a nonnegative even integer j and an integer $k \geq 3$. Roughly speaking, the left hand side is a space of half-integral weight Siegel cusp forms, and the right hand side is a space of those of integral weight.

While, van Hoften gave another lifting theorem of modular forms similar to the above one. It is expected that there is a compatibility between them via theta correspondence. I will study it representation theoretically.

Endoscopic classification for rigid inner forms of classical groups

Endoscopic classification is a classification theory of admissible representations of groups over local fields and automorphic representations which was established by Arthur. In the paper [Is3], I studied the endoscopic classification for inner forms of odd special orthogonal groups. Next, I will study the theory for symplectic groups and even orthogonal groups. Unlike the case of the odd special orthogonal groups, we should consider rigid inner twists when we study these groups. Hence, I expect that the theory for these groups is more complicated than for odd special orthogonal groups. I plan to work with Kakuhama, who is an expert in rigid inner twists.