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Plan of future research

In future research, the goal remains to elucidate the properties of discrete quandles. In this regard, two main challenges have been listed:

1. There are few discrete quandles with known explicit structures.
2. There are very few known invariants that serve as criteria for classifying discrete quandles.

In my previous research, I defined quandles generalizing the fundamental quandle of hyperbolic knots from discrete groups acting on hyperbolic spaces and showed that these are discrete subquandles of homogeneous spaces related to hyperbolic spaces. Just as with the complexities of hyperbolic manifolds, analyzing the quandles defined in this way involves many difficulties. However, what was previously abstract can now be handled somewhat more concretely. On the other hand, while have introduced the Euler characteristic for quandles against the backdrop of compact symmetric spaces, this is an invariant intended for finite quandles. Hence, there is a need for invariants applicable to infinite discrete quandles. Therefore, I considered introducing hyperbolic geometric methods, particularly coarse geometry methods, into quandle theory. So far, I have obtained foundational results for applying coarse geometry to quandles, and in future research, I aim to deepen this approach. The following points are listed as research objectives:

A) Construct various discrete quandles geometrically and elucidate their specific properties.
B) Based on concrete analytical results, construct invariants for discrete quandles and classify them.
C) Apply the results and methods obtained through this process to knot theory and the theory of symmetric spaces.

Research policy

This research aims to classify discrete quandles, especially countably infinite quandles, by analyzing their algebraic structures using geometric methods. Currently, there are few examples of countably infinite quandles with fully elucidated structures. However, focusing on 3-dimensional hyperbolic geometry and related properties of knots, I have constructed a family of concrete countably infinite quandles. By replacing the 3-dimensional hyperbolic space with more general non-compact symmetric spaces, the methods used can potentially be applied further, aiming for the construction of various discrete quandles as outlined in research objective (A).

Additionally, by referencing the properties of the constructed discrete quandles, I will study the properties of various invariants as outlined in objective (B). In particular, through collaborative research with Iwamoto and Kodama, we found that quasi-isometric invariants in coarse geometry can serve as invariants for quandles. By studying the fundamental invariants in coarse geometry,

such as ends and growth rates, I aim to clarify the relationship between group invariants and quandle invariants.