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Summary of research results to date

Background of the research

A quandle is an algebraic system that was independently introduced as knot invariants by Joyce (1982) and Matveev (1982). These structures can be regarded as an abstraction of the conjugate operation in groups. In recent years, quandles have been studied from various perspectives, including knot theory and Hopf algebras. Notably, symmetric spaces become quandles through their point symmetries, providing important geometric insights into the algebraic structure of quandles.

In the study of algebraic structures, exemplified by groups, there is often a progression of research from finite to continuous, and then to discrete structures. Finite quandles have been the focus of intensive study recently, particularly in relation to knot theory. Quandles with topological structures, or more strongly, with differentiable structures, have been studied as symmetric spaces and their generalizations. Following this natural progression, we have been studying discrete quandles, particularly quandle structures on countable or at most countable sets, from a geometric standpoint.

Research objective

The fundamental quandle defined for a knot is a powerful invariant that determines the weak equivalence class of the knot. However, the fundamental quandle is an abstract object expressed via generators and relations and is often a countably infinite set. Analyzing such discrete quandles is generally difficult. Moreover, the algebraic structure of quandles is complex, and there are not many examples for which the structure has been concretely elucidated. In general, it is often challenging to distinguish abstract quandles. Therefore, the research aims to construct invariants for discrete quandles and quandles from a geometric perspective.

Previous research results

In our previous research, we have obtained the following results:

1. We explicitly described the hyperbolic structures, particularly the formula for hyperbolic volume, of a class of links (Paper [1]).
2. We constructed discrete quandles from the perspective of 3-dimensional hyperbolic geometry, which is well-suited for knot theory (Paper [2]).

3. We defined the Euler characteristic for quandles based on properties of Riemannian symmetric spaces (Paper [3]).
4. We introduced a graph structure and a metric for quandles inspired by geometric group theory (Paper [4]).
5. We constructed quandle extensions using oriented Grassmann manifolds (Paper to be presented [5]).
6. We constructed quandles that encompass any finite quandle (Paper to be presented [6]).
7. We provided a method for determining the non-embeddability of quandles using links (Paper to be presented [7]).

Furthermore, as a future prospect, we aim to find applications for the specific quandles and invariants obtained from our previous research, focusing on knots and symmetric spaces.