

Research Plan

Taizo Kanenobu

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1 Enumeration and classification of ribbon 2-knots

We continue to enumerate and classify ribbon 2-knots.

1. We enumerate and classify the ribbon 2-knots of 1-fusion with length up to 7.
2. We enumerate and classify the ribbon 2-knots with ribbon crossing number 5. In this family there exist ribbon 2-knots of 2-fusion. So, we might need some new technique to classify them.

2 Classification of ribbon knots with symmetric union presentations

The symmetric union introduced by Kinoshita and Terasaka and its generalization is a well-known method to construct a ribbon knot. Lamm gives many examples of symmetric unions and asks whether every ribbon knot is a symmetric union representation. Besides this, Eisermann and Lamm introduced a notion of symmetric equivalence among symmetric union diagrams. We would like to consider a classification of knots presented as symmetric union. We can find several examples sharing the same polynomial invariants such as the Alexander, Conway, Jones, HOMFLYPT, Q , or Kauffman polynomials. For a particular family of knots we have difficulty in classifying them.

3 Factorizable Jones polynomials of knots

The Jones polynomial of a composite knot is the product of the Jones polynomial of each factor knot. However, there exists a prime knot whose Jones polynomial is the product of the Jones polynomial of some knots. For example, infinitely many prime knots whose Jones polynomials are the square of that of the figure-eight knot have been constructed. Also, 2-bridge knots whose Jones polynomials are product of the Jones polynomial of the figure-eight knot and that of other 2-bridge knot are constructed. Moreover, there are such knots with small crossing number; there are 11 prime knots with crossing number up to 10 whose Jones polynomial are the product of two Jones polynomials of some knots.

We study factorizations of Jones polynomials of prime knots. My collaborator Toshio Sumi began with a computer analysis of prime knots with at most 18 crossings, finding there are 436,236 non-trivial factorizations amongst 58,021,795 knots.