Summary of Recent Research Results

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1 Meridional epimorphisms between ribbon 2-knot groups

A 2-knot is an embedding of a 2-sphere in the 4-space. In particular, a ribbon 2-knot is a 2-knot that bounds a ribbon 3-disk, which is an immersed 3-disk with only ribbon singularities. The ribbon crossing number of a ribbon 2-knot is the minimal number of the ribbon singularities of any ribbon 3-disk bounding the knot. A ribbon 2-knot is also constructed by adding r 1-handles to a trivial 2-link with $r+1$ components for some r. The least number of r is called the fusion number.

For classical knot groups, some authors have been studying an epimorphism preserving a peripheral structure. We studied a similar problem for ribbon 2-knots, that is, we examined the existence of an epimorphism preserving a meridian, which we call a meridional epimorphism.

Tomoyuki Yasuda enumerated the ribbon 2-knots with ribbon crossing number up to four, and the author classified them. Among them there are a pair of ribbon 2-knots, which have isomorphic groups and have not been decided whether they are isotopic or not. Excluding one of this knot, there are 121 ribbon 2-knots of 1-fusin with crossing number up to four.

Among the 14,520 pairs of the groups of these ribbon 2-knots, we examined the existence of a meridional epimorphism. First, using the Alexander polynomial, representations to $\mathrm{SL}(n, \mathbb{F})$ with $n = 2, 3$ and $\mathbb{F}$ a finite field, and the corresponding twisted Alexander polynomial, we showed 34 pairs of ribbon 2-knots could have a meridional epimorphism. Furthermore, using the commutator subgroup, we showed the 2 pairs do not have a meridional epimorphism. Then, we construct meridional epimorphism for 27 pairs. Thus, we cannot decide the existence of a meridional epimorphism for 5 pairs; considering chirality, this is essentially 3 pairs.

This is joint work with Toshio Sumi (Kyushu University).

2 Jones polynomial of a knot with small span

We have decided the Jones polynomials of knots with span up to 4 and enumerate the potential Jones polynomials of knots with span 5 and 6. The span of a Laurent polynomial is the difference between the highest and lowest degrees. We use the evaluations of the Jones polynomial due to Jones and Lickorish-Millett. For the polynomials with span ≥ 7 we have shown there exist infinitely many polynomials with span n satisfying these evaluations.

This is joint work with Kengo Kishimoto (Osaka Institute of Technology) and Toshio Sumi (Kyushu University).