Research achievements to date (Atsushi Katsuda)

My research has been in Riemannian geometry, spectral geometry, and dynamical systems; in recent years I have also become interested in representation theory and integrable systems.

(1) **Convergence theorems for families of Riemannian manifolds** Global Riemannian geometry began to develop in earnest around 1950. Until the 1970s the main approach was to compare individual Riemannian manifolds, such as standard spheres, with one another. From the late 1970s, following Gromov, the viewpoint of studying "entire classes of Riemannian manifolds" using the Gromov-Hausdorff distance was introduced. One of the central early results, Gromov's compactness (convergence) theorem, had a proof that was not fully understood at the time. I provided a detailed proof of that theorem. I believe this work helped trigger later developments, including Kenji Fukaya's collapse theory.

(2) **Distribution of primitive closed orbits in hyperbolic dynamical systems (Abelian extensions)** Guided by Toshikazu Sandata's program of "number-theoretic methods in geometry," I studied primitive closed orbits in hyperbolic dynamical systems. Connecting symbolic dynamics and its thermodynamic formalism, the work led to density theorems for infinite Abelian groups (joint work with Sandata) that go beyond direct analogues of number-theoretic results.

(3) **Spectral geometry of graphs** On spectral geometry of graphs I collaborated with Hajime Urakawa on a discrete version of the Faber-Krahn inequality, and with Hironobu Fujii on constructions of isospectral graphs.

(4) **Inverse spectral problems for Riemannian manifolds with boundary** This inverse problem is the one Gel'fand posed at the 1954 ICM: reconstruct a manifold with boundary from spectral data (the eigenvalues of the Laplacian under Neumann boundary conditions and the boundary values of eigenfunctions). Belishev and Kurylev solved this using the boundary control method. In realistic situations the spectral data are often only partial and contain errors, so one must study stability. Ultimately, results obtained in collaboration with Anderson, Kurylev, Lassas, and Taylor were assembled to address this stability question.

(5) **Isometry groups of Riemannian manifolds** Bochner's classical result says that for a compact Riemannian manifold with nonpositive Ricci curvature, the dimension of its isometry group is at most the dimension of the manifold, with equality if and only if the manifold is a flat torus; moreover, if the Ricci curvature is negative the isometry group is finite. I carried out joint work with graduate students and collaborators (with Takuya Nakamura) on the stability of this statement, and with Takeshi Kobayashi on concrete estimates and extensions of isometry groups under the latter curvature conditions. For the stability result we obtained substantially stronger conclusions than initially expected.

(6) **Floquet-Bloch theory for discrete polynomial growth groups** I have long considered non-commutative generalizations of the results obtained in (2). Around 2000 I found a clue but ran into technical difficulties. Only in recent years have I been able to consolidate results for discrete polynomial growth groups. The work links the representation theory of historically difficult non-type I discrete groups with the representation theory of their Malcev completions (Lie groups), and, via representation-theoretic alternative proofs of asymptotic expansion formulas that had been derived using microlocal analysis, extends the range of applicability. These new arguments form the basis of the recent results.