Further studies.
Based on existing studies, the researcher would like to challenge the following.

1. **Nice smooth functions with given Reeb graph and level sets (in the differentiable situation), classifications of Morse-Bott functions etc.**
Based on achievement of the researcher, he plans to study the following.
1-1. **Morse-Bott functions on 3-dimensional closed manifolds.**
1-1-1. **Classifications of Morse-Bott functions on 3-dimensional closed and orientable manifolds of a certain nice class.**
The researcher has studied Morse-Bott functions on connected sums of torus bundles over the circle, lens spaces, and $S^1 \times S^2$ whose level sets containing no critical points consist of S^2 and $S^1 \times S^1$ and which satisfy some mild conditions. Related to this, he has investigated a study on Morse functions to each vertex of whose Reeb graphs exactly one critical point is mapped by Michalak (https://arxiv.org/abs/2403.02291). The fundamental group of a space is a (non-commutative) group representing loops in an algebraic way and in the preprint, relations between the fundamental group of the manifold of the domain and the number of vertices of degree 2 are explicitly studied. Morse functions on circle bundles over closed orientable surfaces are explicitly studied. Topology of level sets are not referred to in the study.

- For Morse functions on connected sums of torus bundles over the circle, lens spaces, and $S^1 \times S^2$, we investigate the pair $(g_f, e_{f_{g_f}})$ of the minimal genus g_f for closed and connected surfaces appearing as components of level sets with no critical point and the number $e_{f_{g_f}}$ of edges for such components. We classify them in the case the pair is minimum in the lexicographical order.
- We consider the previous problem for circle bundles over closed and orientable surfaces.

1-2. **Higher dimensional cases.**
First, it is important to study the case of 4-dimensional closed and orientable manifolds. It is difficult to formulate problems in general dimensions. The researcher has been studying decompositions of manifolds compatible with Morse functions, so-called generic smooth maps into the plane: so-called Heegaard splittings of 3-dimensional manifolds and higher dimensional versions, trisections (multisections). Related knowledge and understandings must be helpful.

2. **Real algebraic versions of the previous study "1." and real algebraic maps respecting the canonical projections of the unit spheres and moment maps on complex projective spaces and more generally toric-symplectic manifolds.**
Explicit studies and contribution to general theory is important. For example, related studies are started by the researcher first in the world as in our "Previous studies".