Previous studies.

The researcher is specialized in "Singularity theory of differentiable maps" and has studied explicit Morse functions, more general nice smooth functions and higher dimensional versions, mainly. The following are main achievement.

1. On differentiable manifolds Morse functions exist densely. Critical points know homology groups and homotopy, information on deformation. Such theory is generalized to higher dimensions. Thom and Whitney started, Levine and later Eliashberg have studied more, and recently Saeki and Sakuma are studying actively. The class of fold maps is a class of smooth maps locally represented as the product maps of Morse functions and identity maps: the canonical projection of the unit sphere gives a simplest example with its set of singular points being the equator and its image is an embedded sphere. We have introduced round fold maps as generalizations, investigated fundamental invariants of the manifolds, and constructed non-trivial examples on manifolds such as ones represented as connected sums of total spaces of bundles over spheres whose fibers are spheres ("Articles" 1.1, 1.2, 2.1, 3: hereafter "Articles" will be omitted). We have also classified codimension -1 round fold maps (1.5). We have also checked that a 3-dimensional closed, and orientable manifolds admit such a map into the plane if and only if it is a so-called graph manifold (1.7). Morse functions with exactly two critical points on homotopy spheres and canonical projections of spheres are generalized to special generic maps. Burlet and de Rham defined such maps in 1970s and Saeki and Sakuma are studying actively. These maps restrict the topologies and the differentiable structures strongly. Elementary manifolds such as elementary manifolds above admit natural special generic maps in considerable cases. We have investigated cohomology rings of manifolds admitting such maps: mainly we have seen non-existence of such maps for projective spaces (4.1, 4.2, 4.4-7, 4.14).

2. The Reeb graph of a smooth function is the quotient map consisting of all components of level sets. In the case of smooth functions on closed manifolds with the sets of all critical values being finite, they are naturally graphs. In the 20th century, it has appeared and it simplifies the manifold. Such graphs are also important in visualization. In 2006 Sharko has asked " Can we have nice smooth functions whose Reeb graphs are given graphs?" . Sharko, followed by Saeki and his student Masumoto and their refinement, has constructed nice smooth functions on closed manifolds whose Reeb graphs are given graphs. Later, Michalak has constructed Morse functions such that components of level sets with no critical point are spheres for suitable graphs. We have studied cases where topologies of components of level sets are as prescribed which may not be spheres or compact (1.3, 1.4, 1.8, 4.3, 4.11, 4.12). We have also studied classifications of Morse functions on 3-dimensional closed manifolds represented as connected sums of the products of the circle and spheres and so-called lens spaces via Reeb graphs and topologies of level sets with components containing no critical point being spheres or tori. This is a higher dimensional version of Michalak's study above on the surface case and improves Saeki's characterization of such manifolds via such functions in 2006, by additional local observations (4.10). As a challenging case, we are studying real algebraic cases (1.6, 2.1, 4.8, 4.9, 4.13).