Nobukazu Kowaki

The applicant has so far investigated toric degenerations of Grassmannians through the framework of tropical hyperplane arrangements. Building on this work, the applicant aims to reinterpret and describe tropical hyperplane arrangements from perspectives originating in other areas, such as the theory of integrable systems. Because tropical geometry interacts with algebraic geometry, integrable systems, and optimization theory, it has the potential to provide a new unifying viewpoint that connects these fields.

While the applicant's main achievements during the doctoral program concern the Grassmannian Gr(3, n), which corresponds to two-dimensional tropical hyperplane arrangements, the next goal is to extend these results to Gr(4, n) and to clarify the structure of tropical hyperplane arrangements of dimension three or higher. As a first step toward this goal, the applicant intends to examine whether the notion of the characteristic polynomial, which plays a central role in classical hyperplane arrangement theory, can be applied or generalized to tropical hyperplane arrangements.

In the two-dimensional case, the characteristic polynomial can be defined in a natural way. However, it remains unknown whether this tropical characteristic polynomial agrees with the characteristic polynomial of the original classical hyperplane arrangement in general. Clarifying this relationship is essential for understanding which kinds of information are preserved and which are lost during the tropicalization process. If the two polynomials do not coincide, the applicant aims to characterize the conditions under which this discrepancy arises.

Because classical algebraic geometry and tropical geometry are closely related, as demonstrated for example by Kapranov's theorem, this investigation can be viewed as an attempt to determine to what extent the information contained in a classical hyperplane arrangement is inherited by its tropical counterpart. Through this direction of research, the applicant seeks to make the correspondence between classical and tropical geometry more explicit.

The applicant also plans to study matching fields and their associated polytopes. These polytopes share similarities with matroid polytopes, and matching fields themselves exhibit matroid-like structures in tropical linear algebra. Since matroids are deeply connected with classical hyperplane arrangements, it is natural to explore whether one can construct a classical hyperplane arrangement from the polytope associated with a matching field, and whether such a construction is related to the corresponding tropical hyperplane arrangement.

By addressing these problems, which lie at the intersection of tropical geometry, hyperplane arrangement theory, matroid theory, and integrable systems, the applicant aims to uncover new mathematical structures underlying tropical hyperplane arrangements and to reconstruct them from a broader viewpoint that incorporates insights from applied mathematics as well.