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Grassmannians are generalizations of projective spaces and have long been studied as one of the most fundamental examples of projective varieties. In recent years, their importance has further increased through connections with areas such as positive geometry and theoretical physics, including particle physics.

One classical approach to studying projective varieties and other algebraic varieties is to analyze their coordinate rings. However, in general, the study of arbitrary quotient rings is extremely difficult. To address this, techniques involving initial algebras—obtained by introducing monomial or weighted orders to reduce ideals to more tractable objects such as monomial ideals or toric ideals—have been actively developed. If the initial algebra satisfies desirable properties such as Cohen–Macaulayness or normality, it is known that the original coordinate ring inherits these properties. Conversely, even when the original coordinate ring enjoys such good properties, the initial algebra does not necessarily inherit them. Thus, there is strong motivation to find as many good initial algebras as possible.

With this perspective, the applicant has been working on obtaining a large class of toric degenerations of Grassmannians. Concretely, the goal is to impose suitable orders on the defining ideal so that it degenerates to a toric ideal. Among the various approaches to toric degenerations of Grassmannians, the applicant focuses on those arising from coherent matching fields, which introduce orders induced by certain matrices. For such an order, determining whether the initial ideal of the Plücker ideal—the defining ideal of the Grassmannian—is toric reduces to verifying that the Plücker coordinates form a SAGBI basis of the Plücker algebra, the coordinate ring of the Grassmannian. A SAGBI basis is the subalgebra analogue of a Gröbner basis.

However, determining SAGBI bases is far from straightforward. Whereas Gröbner bases can be computed in finite time via Buchberger's algorithm, SAGBI bases may be infinite even for finitely generated subalgebras. For this reason, when a coherent matching field is given, a method is needed to determine whether it yields a toric degeneration without directly checking the SAGBI condition. The applicant obtained a sufficient condition for this by extracting combinatorial information from the tropical hyperplane arrangement associated with the coherent matching field.

More specifically, it is known that if the polytopes associated with two matching fields are combinatorially mutation equivalent—a relation originating in cluster algebra theory—then one admits a toric degeneration if and only if the other does as well. Thus, by analyzing the relationship between the tropical hyperplane arrangements corresponding to two matching fields, the applicant characterized when their associated polytopes are mutation equivalent and thereby obtained the desired criterion.

Combinatorial mutation equivalence arises from cluster algebras, and the above line of research can be interpreted as extracting cluster-algebraic information from tropical geometric data. Similar phenomena are known in the study of integrable systems. Motivated by this, the applicant is currently investigating whether the results described above can be reformulated in the language of infinite-dimensional integrable systems, with the aim of uncovering new integrable structures underlying the toric degenerations of Grassmannians.