(2) Study proposal Makiko Mase

(i) Aim of the study

The aim of the study is to understand algebro- and arithmetic geometric aspects of algebraic complex $K3$ surfaces, which we simply call $K3$ surfaces otherwise noted. The study is deeply involved in classical areas such as singularity theory, automorphism groups, and algebraic curve theory, and in modern area of mathematical physics. For our purposes, we are expected to analyze certain lattices associated to $K3$ surfaces together with symplectic automorphism actions on them, and topological characteristics of them as well as relations with hypergeometric functions and monodromies.

Problems

1. $K3$ surfaces admitting symplectic automorphisms.
2. Mirror dualities of families of $K3$ surfaces.
3. Double covering structure on a weighted projective plane.
4. $K3$ surfaces admitting non-symplectic automorphisms.
5. Weierstrass semigroups of pointed curves and $K3$ surfaces.

(ii) Study methods

Problem 1 Let X be a $K3$ surface admitting a finite group G acting symplectically on X, and $L := L_G$ associated to the resolution Y of the singularities in X/G. Our aim is to determine whether or not the minimal primitive closure $\tilde{L}$ of L is uniquely determined. We attack this problem by studying the discriminant group.

Problem 2 We determine whether or not the lattice duality between families of $K3$ surfaces extend to transpose duality due to S. Tanabe, and dualities in terms of modular forms due to N. Yui and her corraborators.

Problem 3 If there exists a divisor D on a weighted projective plane $\mathbb{P}$ such that $2D$ is Cartier, then, there exists a double covering $X \to \mathbb{P}$ with branch divisor $2D$ and X is obtained as $\mathrm{Proj}(\mathcal{O}_{\mathbb{P}} \oplus \mathcal{O}_{\mathbb{P}}(D))$. The aim of this topic is to study the Weierstrass semigroup of algebraic curves on the algebraic variety X associated to this double covering.

Problem 4 (A joint work with Professor Takeshi Takahashi in Niigata University) There are pairs $(\mathbb{P}^2, B)$ with a plane curve B, which are called $K3$-orbifold classified by A.M.Uludağ. We are interested in constructing explicitely a projective $K3$ surface X together with a finite group G that can realize a $K3$-orbifold.

Problem 5 (A joint work with Professor Jiryo Komeda in Kanagawa Institute of Technology) We are interested in studying which Weierstrass semigroups for pointed curves are on/off a $K3$ surface, and we'd also like to give a characterisation for Weierstrass semigroups attained by an algebraic curve on a $K3$ surface that is the triple covering of a rational surface.

(iii) Aspects

We expect as a consequence that we can understand algebraic and arithmetic properties of geometry of $K3$ surfaces. We hope to extend our area of study for non-projective $K3$ surfaces : automorphism groups, behaviour of the periods.