Research plan (Mikiya Masuda) December, 2025

I would like to regard toric topology widely as mathematics about the topology and geometry of torus actions and related combinatorics and extend and deepen this area. Currently, I am working on the following projects.

(1) Research on mathematics related to Hessenberg varieties which are subvarieties of flag varieties. Together with Hiraku Abe, Megumi Harada, Tatsuya Horiguchi, I worked on the cohomology rings of Hessenberg varieties during this decade. It turned out that this work is related to hyperplane arrangements and Stanley- Stembridge conjecture on graph theory. My goal is to solve the Stanley-Stembridge conjecture and I have been working with Takashi Sato for three years to aim the solution of the conjecture. So far, we obtained results on

(1) Characterization of regular semisimple Hessenberg varieties with cohomology rings generated in degree 2.
(2) Relation between the twins of regular semisimple Hessenberg variety and unicellular LLT polynomials.
(3) Automorphism groups of regular semisimple Hessenberg varieties.

Recently, I found that the modular law for regular semsimple Hessenberg varieties can be proved in terms of GKM theory (joint work with Sato and Horiguchi). Combining our work with a result by Abreau-Nigro (the proof is also elementary), we can obtain an alternative proof Shareshian-Wachs conjecture first proved by Brosnan-Chow. Their proof uses highly advanced algebraic geometry. I am planning to deepen the research in this direction and reach the solution of the (graded version of) Stanley-Stemberidge conjecture. The original (non-graded version of) Stanley-Stembridge conjecture has been affirmatively answered by Hikita a year ago.

(2) Tolman constructed T^2-action on a closed 6-manifold which is Hamiltonian but not K$\acute{}$ahler. This action has 6 fixed points, so it would be meaningful to investigate how exotic this action is. For that, I (jointly with Donghoon Jang, Shintaro Kuroki, and Takashi Sato) classified almost complex T^2-action on closed 6-manifolds with 4 fixed points. Based on this research, I would like to classify almost complex T^2-actions on closed 6-manifolds with 6 fixed points.