Future Research Plan

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The objective of this research is to rigorously formulate the mathematical structures inherent in the Lorentzian path integral of quantum gravity from the perspectives of complex analysis and asymptotic analysis, thereby reconstructing fundamental problems of quantum gravity —such as spacetime singularity resolution and cosmogenesis—as ”complex geometric problems.” Specifically, for gravitational theories including general relativity, the applicant aims to develop non-perturbative analyses based on resurgence theory and Picard-Lefschetz theory to achieve a unified understanding of physically allowed spacetime structures and their transition amplitudes in quantum gravity. The following two research themes will be pursued.

1. Non-perturbative Structure and Singularity Analysis in Quantum Gravity using Resurgence Theory

Path integrals in quantum gravity generally diverge, and their saddle-point structures are extremely complex. This theme aims to establish a methodology for rigorously evaluating non-perturbative quantum gravity effects by utilizing complex gradient flows via Picard-Lefschetz theory and Borel summation via resurgence theory. In previous work, the applicant applied these methods to the path integral of a homogeneous and isotropic universe model and clarified that Vilenkin’s tunneling proposal emerges naturally as a contribution from non-perturbative complex saddle points [6]. This represents an attempt to replace the conventional physical requirement of ”choosing boundary conditions for the wave function of the universe” with the mathematical requirement of ”choosing the complex integration contour.”

This plan involves extending this methodology to anisotropic Bianchi spacetimes and spacetimes exhibiting chaotic behavior. In particular, by analyzing the Stokes phenomena between saddle points in complex phase space, the applicant aims to elucidate the mathematical necessity behind the selection of uniform and isotropic spacetime structures. Furthermore, regarding the complexification of singularities and the information loss paradox in black hole spacetime path integrals, this research aims to achieve a mathematical understanding and resolution from the perspective of quantum gravity by describing the saddle-point structure of the path integral within a complete resurgent framework.

2. Mathematical Formulation and Consistency Analysis of Lorentzian Quantum Gravity

This research systematically elucidates the quantum nature of spacetime based on the Lorentzian path integral. The applicant has previously shown that the contribution of complex saddle points and the choice of integration contours essentially dictate the physical interpretation of quantum cosmology. This study will evolve this framework into higher-dimensional gravity theories and braneworld models, re-examining cosmogenesis, singularity problems, and holographic entropy from the standpoint of Lorentzian path integrals based on complexified spacetime geometry.

A key focus is determining which complex saddle points are physically meaningful and actually contribute to the quantum amplitude. Previous research in Jackiw-Teitelboim gravity suggests that wormhole-type saddles, usually considered non-contributing, may play a crucial semi-classical role under specific analytic continuations or contour choices. This study will re-evaluate these findings from the perspective of Lorentzian path integrals, systematically analyzing the relationship between complex saddle structures, Stokes phenomena, and physical interpretations to provide a consistent formulation of quantum cosmology.

Furthermore, this research focuses on the relationship between quantum cosmology and holographic entropy. In braneworld models, the entropy of the cosmological horizon generally cannot be described by a simple area formula; a consistent understanding with the variational principle of the action and the path integral is indispensable. Therefore, the applicant will re-examine the relationships between Wald-type entropy and holographic entanglement entropy to clarify fundamental quantities such as the creation probability and entropy of the universe within the Lorentzian quantum gravity framework. Through these efforts, this research aims to strengthen the mathematical foundation of quantum cosmology and pioneer new theoretical developments in Lorentzian quantum gravity.