Summary of Research Achievements [2]

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To provide a mathematically and physically rigorous elucidation of fundamental questions in modern cosmology—specifically, cosmogenesis and the problem of spacetime singularities—the applicant has conducted theoretical research primarily focused on constructing cosmological creation models based on quantum gravity and the rigorous evaluation of their path integrals. The major achievements are summarized below.

Rigorous Evaluation of Cosmogenesis Models Based on the Path Integral Formulation of Quantum Gravity

Regarding the origin of the universe, the "quantum cosmogenesis" hypothesis–which posits that the universe was created from "nothing" (a state without spacetime) via quantum effects–is a leading candidate. The "no-boundary proposal" (1983) by Stephen Hawking and James Hartle is well-known as a representative framework for this. Conventionally, this proposal has been formulated using Euclidean path integrals in quantum gravity; however, debates regarding its theoretical validity have persisted for decades due to issues such as the non-positive definiteness of the action and ambiguities in the choice of integration contours.

In recent years, a more rigorous formulation based on Lorentzian path integrals, which maintain real time, has attracted significant attention. The applicant re-examined the validity of the no-boundary proposal and Alexander Vilenkin's "tunneling proposal" from first principles using this Lorentzian path integral approach [6]. To address the theoretical ambiguities unavoidable in conventional methods, the applicant introduced mathematical techniques such as resurgence theory and Picard-Lefschetz theory, successfully resolving the ambiguities associated with the evaluation of the path integral. Consequently, the applicant demonstrated that the wave function of the universe aligns with the prediction of the tunneling proposal rather than the no-boundary proposal, providing a definitive conclusion to a long-standing theoretical debate in quantum cosmology [6]. Furthermore, by analyzing linear perturbations in a homogeneous and isotropic background, the applicant revealed that the realization of a uniform and isotropic spacetime is probabilistically difficult in the context of quantum cosmogenesis [5, 13]. Other contributions include research on quantum tunneling effects applying Picard-Lefschetz theory [16], the path integral formulation of DeWitt boundary conditions [11], and the formulation of the no-boundary proposal within Jackiw-Teitelboim gravity and Hořava-Lifshitz gravity [3, 8].

Elucidation of Singularity Problems in the Early Universe and Black Hole Interiors via Quantum Gravity

The applicant has re-examined the singularity problems in the early universe and black hole interiors from the perspective of quantum gravity [14, 15, 2]. Regarding the initial singularity of

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[2] Reference numbers correspond to the serial numbers in the separate "Publication List."

the universe, the validity of the DeWitt boundary condition (the condition that the wave function vanishes at the singularity) was investigated. It was shown that within the framework of general relativity, the spacetime wave function breaks down perturbatively under this condition when tensor perturbations are considered [15]. Conversely, in Hořava-Lifshitz gravity, this problem is resolved, yielding a stable DeWitt wave function; in particular, exact analytical solutions were obtained in the anisotropic scaling limit [14, 15].

Additionally, noting that the interior of a static, spherically symmetric black hole can be viewed as a homogeneous but anisotropic universe described by the Kantowski-Sachs metric, the applicant constructed the Wheeler-DeWitt equation for this system. By analyzing the system while varying the parameter controlling the strength of quantum effects (the gravitational constant), it was proven that in regions where quantum effects are strong, the wave packet of the wave function departs from the classical trajectory, exhibiting behavior that suggests singularity avoidance [2]. Furthermore, an analysis using an internal clock showed that stronger quantum effects lead to a longer time until singularity formation, presenting the possibility of singularity suppression by quantum gravity [2]. These results clearly demonstrate that quantum gravity based on the Wheeler-DeWitt equation can alleviate singularity problems in two distinct contexts: the early universe and black hole interiors.