Research Plan

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Research on Three-Dimensional Almost Contact Metric Manifolds Using the Newman-Penrose Formalism

Background and Objectives: In 2025, the applicant reformulated aspects of the theory of three-dimensional almost contact metric manifolds using a three-dimensional Riemannian version of the Newman-Penrose (NP) formalism. At the outset, this reformulation was expected to be no more than a reinterpretation of existing methods and not to yield any genuinely new insights. However, the resulting framework turned out to be more transparent than anticipated, and this improved clarity made it possible to solve a previously unresolved classification problem for η-Einstein manifolds under the assumptions of compactness and normality. While the NP formalism does not provide strictly more information than conventional formulations, it reorganizes the available information in a way that significantly improves conceptual clarity. This reorganization proved to be advantageous in obtaining the above result. These findings indicate that the NP formalism has sufficient potential to serve as an effective tool for the study of three-dimensional almost contact metric manifolds. Motivated by this observation, the applicant aims to investigate a broader range of problems in three-dimensional almost contact metric geometry using the NP formalism. Several aspects of three-dimensional almost contact metric structures remain poorly understood. In addition, the relationship between these structures and Ricci solitons continues to be an active area of research, and this is another domain in which the NP formalism is expected to be particularly effective.

Significance: The applicant' s previous research has primarily focused on applications of contact geometry to relativity. Since the Newman-Penrose formalism itself originated in relativity, demonstrating its effectiveness in the analysis of three-dimensional almost contact metric structures would also highlight the significance of the reverse direction, namely, applications of relativistic techniques to contact geometry. Such a perspective is meaningful not only within contact geometry but also as a contribution to mathematical relativity.

Methodology: There are two main approaches to the Newman-Penrose formalism: a vectorial approach and a spinorial approach. In general, the amount of information obtained from these two approaches does not differ substantially. However, the spinorial approach tends to provide slightly richer information in certain special situations. Moreover, even in general settings, it often offers technical advantages in the classification of the Ricci tensor via Petrov types and in the analysis of conformal structures through the Cotton – York tensor. In the applicant' s 2025 work, the vectorial approach was employed. In the present project, the applicant plans to switch to the spinorial approach and to use it to further investigate the geometric properties of three-dimensional almost contact metric structures.