Research Results

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A contact manifold is an odd-dimensional manifold equipped with a one-form known as a contact form. Sasakian manifolds, a subclass of contact manifolds, have appeared in various fields of mathematical physics, most notably in the context of the AdS/CFT correspondence. Since obtaining my PhD, I have been conducting research under the theme of "Contact Geometry and General Relativity." I hereby report my research achievements and status for the year 2025.

1. Geometric Properties of Sasaki-Quasi Killing Spinors on three-Dimensional Pseudo-Riemannian Manifolds

Aiming to clarify the properties of Sasaki-quasi Killing spinors on three-dimensional pseudo-Riemannian Sasakian manifolds, I authored a paper on this subject in 2024. This paper was successfully published in the Journal of Geometry and Symmetry in Physics in 2025.

2. Study of three-Dimensional Almost Contact Metric Manifolds via the Newman-Penrose Formalism

The Newman-Penrose (NP) formalism is a well-known method for analyzing four-dimensional spacetimes, particularly effective for investigating spacetimes that admit a shear-free geodesic null congruence (Robinson spacetimes). In 2015, Aazami proposed an analogous method for three-dimensional Riemannian manifolds. I discovered that in three-dimensional almost contact metric manifolds, the "normality" condition is equivalent to the existence of a shear-free geodesic. This finding suggested that the 3D NP formalism would be highly effective in this context. By translating various concepts of traditional almost contact geometry into the 3D NP formalism, I provided concise proofs for several existing results. Furthermore, regarding the classification problem of η-Einstein manifolds within 3-dimensional normal almost contact metric manifolds—an open problem since the 1980s—I succeeded in solving this for the compact case. Although several preceding studies addressed this problems, this result had not been achieved. Translating the problem into the NP formalism organized the information in a way that facilitated the solution. A preprint is currently in preparation.

3. Construction of Non-Uniform String Cloud Spacetimes Using 3-Dimensional Almost Contact Metric Manifolds A collection of Nambu-Goto (NG) strings continuously distributed in spacetime is called a string cloud. A string cloud spacetime is a solution to the Einstein equations where a string cloud serves as the matter field. While previous studies on string cloud spacetimes often focused on general theoretical frameworks, I, through joint research with Prof. Ishihara and his collaborators, discovered a systematic method to construct string cloud spacetimes using three-dimensional almost contact metric manifolds. By arranging NG-strings along the Reeb flow of a three-dimensional almost contact metric manifold and lifting them to a four-dimensional spacetime using a shear-free geodesic null congruence (SGNC)—a process known as the Robinson lift—we obtained a string cloud where the SGNC lies on the worldsheet. Moreover, the configuration of this string cloud can be made spatially non-uniform, allowing for the "inhomogenization" of many existing homogeneous spacetimes. For example, we successfully constructed imhomogeneous string cloud versions of the Taub-NUT, Schwarzschild, and Friedmann models. A preprint is currently in preparation.