Research Plan --- Yasuo Matsushita
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My basic and main research is again concerning the geometry on the indefinite metric spaces.

The Research Plan in 2026:

It is known in Matsushita's paper (1988) that a simply connected 4-manifold admits a field of 2-planes, equivalently a neutral metric, if and only if it satisfies the conditions $\tau \pm \chi = 0 \bmod 4$ as Atiyah's condition. Therefore, the connected sum mCP^2＃－nCP^2 admits a field of 2-planes, equivalently a neutral metric, if and only if both m and n are odd. It will be interesting to analyze neutral metrics on these 4-manifolds, since these Euler characteristics and Hirzebruch indices are not zero.

The Research Plan continued from 2024:

The counterexamples of the Goldberg Conjecture for pseudo-Riemannian manifolds in dimensions 6 and 8 have been reported from 2007 until 2017. For manifolds of the lowest dimensional case, dimension 4, the situations concerning the Goldberg Conjecture it is quite different from the manifolds of higher dimensional cases.
Therefore, our research target will be focused on the 4-dimensional manifolds, if the Goldberg Conjecture is true or not.

1. There are several canonical types of Weyl curvatures for indefinite metric spaces, with structure groups SO(p,q). If we consider the geometry of such indefinite metric spaces with their corresponding spinor groups Spin(p,q), then the Weyl curvatures can be classified into further finer canonical types.
2. This approach can be highly expected to analyze the indefinite manifolds with Spin(p,q) deeper than with structure groups SO(p,q).
3. We already published three papers in this direction of spinor approach to neutral geometry, especially to Walker4-manifolds. These papers are written together with Peter R. Law, a PhD student of Roger Penrose in 1983.
4. Walker manifolds are some specific manifolds, which carry a field of parallel null planes. There are, in fact, two kinds of Walker metrics, Types I and II. There are many papers on Type I, but not on Type II. Even though our analysis on Type II is still under primitive stage, we feel a large possibility to approach on Type II, which exhibit quite different aspect than Type I. This analysis can be submitted for publication soon.
5. This may become an initiation to analyze Walker Type II geometry.
6. We will be seeking if the Goldberg conjecture is true or not for four-dimensional neutral manifolds.

Above mentioned are the works to be analyzed immediately. If we consider a Lorentz metric as an operator on a manifold, then it can be a wave operator. Similarly, a neutral metric on a 4-manofold corresponds to an ultrahyperbolic operator, intensively analyzed by Fritz John. It is hoped that relations between the solutions of these second order partial differential equations (PDE's) and the curvature tensors of indefinite metric spaces, from a point of view of Physics.