

In commutative ring theory, various concepts have been defined, and these concepts are actively studied, intertwining with various fields outside of commutative ring theory. Among these, Cohen-Macaulay and Gorenstein properties are particularly important research areas and are the subject of active study.

While assuming Cohen-Macaulay property of the base ring is not a particularly strong assumption, it facilitates various discussions. Therefore Cohen-Macaulay property has been assumed as the foundation of many theories. On the other hand, Gorenstein rings possess beautiful symmetries in the ring itself and in the various mathematical objects derived from them.

Due to these properties, many researchers have conducted extensive research on Gorenstein and Cohen-Macaulay rings, including connections with various fields.

Through this research, many researchers have come to realize that there is a relatively large gap between Gorenstein and Cohen-Macaulay properties. Therefore, in order to bridge this gap, researchers have sought concepts weaker than Gorenstein property but stronger than Cohen-Macaulay property. Among these, concepts such as level property, almost Gorenstein property, and nearly Gorenstein property have been defined and studied. However, these properties do not work well with basic operations on commutative rings, particularly localization, which is perhaps the most fundamental operation in commutative ring theory.

In 2025, when studying the homogeneous coordinate ring of a Schubert submanifold of a Grassmann manifold, I discovered that the radical property of the trace ideal of the canonical module of this ring is close to the ring being a Gorenstein ring. Therefore, I investigated under what circumstances radical property of the trace ideal of the canonical module holds for rings other than homogeneous coordinate rings of Schubert submanifolds of Grassmann manifolds. And I found that it corresponds to a property next to the one that characterizes Gorenstein property in multiple types of rings. Thus, I have named rings with this property canonical trace radical rings and am currently studying their properties. And I plan to continue studying it further.