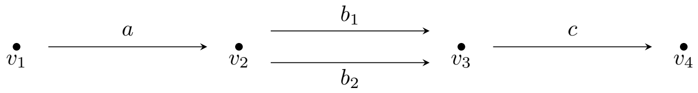
今後の研究計画 (英訳) Fumika Mizoguchi

We defined the nilpotent Lie algebra $\mathfrak{n}_Q$ from a finite quiver without cycles. In the path algebra we used, vertices are treated as paths of length 0. However, the current method for obtaining the nilpotent Lie algebra $\mathfrak{n}_Q$ does not include the vertices as the basis. Therefore, we define a new method to obtain the solvable Lie algebra $\mathfrak{s}_Q$ by adding the vertices as paths of length 0 in a similar manner. Consider the following quiver Q:



Then the Lie algebra $\mathfrak{s}_Q$ is given by $\mathfrak{s}_Q = \mathrm{span}\{v_1, v_2, v_3, v_4, a, b_1, b_2, c, ab_1, ab_2, b_1c, b_2c, ab_1c, ab_2c\}$. The solvable Lie algebra $\mathfrak{s}_Q$ also provides a wide variety of specific examples, similar to the nilpotent Lie algebra $\mathfrak{n}_Q$. We investigate the relationship between the existence of special geometric structures on $\mathfrak{s}_Q$ and combinatorial properties of the quivers.

We clarify the conditions under which the nilpotent Lie algebra $\mathfrak{n}_Q$ and solvable Lie algebra $\mathfrak{s}_Q$ obtained by a finite quiver without cycles admit (A) Ricci-flat pseudo-Riemannian metrics, and (B) symplectic structures. Furthermore, (C) we extend the method of constructing nilpotent and solvable Lie algebras from quivers.

(A) Ricci-flat pseudo-Riemannian metrics

In general, homogeneous pseudo-Riemannian Ricci-flat metrics are not necessarily flat. Nilpotent and solvable Lie groups have examples that allow non-flat Ricci-flat pseudo-Riemannian metrics. Therefore, it is necessary to study when nilpotent and solvable Lie groups admit Ricci-flat pseudo-Riemannian metrics of a given signature. Therefore, we consider the following problem.

Problem. When do the nilpotent Lie algebra $\mathfrak{n}_Q$ and the solvable Lie algebra $\mathfrak{s}_Q$ obtained by a finite quiver without cycles admit Ricci-flat pseudo-Riemannian metric of a given signature?

We obtained the result when the nilpotent Lie algebra $\mathfrak{n}_Q$ obtained by a finite quiver without cycles is of two-step about this problem ([M]). However, the signature of the Ricci-flat pseudo-Riemannian metric obtained from this theorem must satisfy certain conditions, so it cannot be constructed for arbitrary signatures.

(B) Symplectic structures

Nilpotent and solvable Lie groups that admit left-invariant symplectic structures provide many valuable examples. However, in general, it is unclear whether a given nilpotent Lie group admits a symplectic structure. Thus, we consider the following problem.

Problem. When do the nilpotent Lie algebra $\mathfrak{n}_Q$ and the solvable Lie algebra $\mathfrak{s}_Q$ obtained by a finite quiver without cycles admit symplectic structures?

We obtained the result when the nilpotent Lie algebra $\mathfrak{n}_Q$ obtained by a finite quiver without cycles is of two-step about this problem ([M]). On the other hand, for three steps or more, the structure of the quiver becomes more complex, so further investigation is necessary.

(C) Expansion of the method

The nilpotent Lie algebra $\mathfrak{n}_Q$ and the solvable Lie algebra $\mathfrak{s}_Q$ obtained by a finite quiver without cycles contain many specific examples that have not been derived in previous research. On the other hand, there are also Lie algebras that cannot be obtained by this method, such as the five-dimensional Heisenberg algebra. Therefore, we require an extension of the construction method.

References

[M] Mizoguchi, F., "Two-step nilpotent Lie algebras obtained by quivers and geometric structures", In: Geometric and Harmonic Analysis on Homogeneous Spaces and Applications, Springer Proceedings in Mathematics and Statistics, to appear.