A summary of past research achievements

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My research field is Complex Analysis and my main subject to study is the theory of Teichmüller space.

(1) Dynamical system of geodesic flow on hyperbolic manifolds(No.1 and 3 in the list of publication): We proved in [1] by an elementary method that almost all orbits of the geodesic flow are dense in a hyperbolic 3manifolds with finite volume. A theorem on continued fraction approximations is proved in [3].

(2) Hyperbolic Geometry(No.6 and 25) : The paper [6] gives the best possible lower bound of the lengths of closed geodesics with self-intersection on 2-dimensional hyperbolic orbifods.

(3) Outer and inner radii of Teichmüller spaces(No.2,4,5,7 and 8) : We gave in [4] a complete characterization of a Riemann surface whose Teichmüller space defined by the Bers embedding has the biggest possible outer radius.

(4) Complexified lambda lengths as parameters for $SL(2,\mathbb{C})$-representation space of a punctured surface(No. 9,10,15,16,21 and 22) : We introduce complexified lambda-lengths employed by R.C.Penner to parametrize the decorated Teichmüller space of punctured hyperbolic surfaces.

(5) Arithmetic Fuchsian groups(No.12) : We classified arithmetic Fuchsian group with signature $(0; e_1, e_2, e_3, \infty)$.

(6) Weil-Petersson volumes of two dimensional moduli spaces(No.11 and 14) : We calculated the Weil-Petersson volumes of the two-dimensional moduli spaces.

(7) Variations of McShane identities (No.17 and 20) : We gave two variations of McShane identity. McShane identity played an important role in Mirzakhani's recurrence formula for the Weil-Petersson volumes of moduli spaces.

(8) Global coordinate-systems for Teichmüller spaces using lengths of closed geodesics (No.18,19 and 23) : Lengths of a finite number of closed geodesics determine a hyperbolic structure on a surface of finite topological type and hence paramerize the Teichmüller space of the surface. We found the least possible number of closed geodesics whose lengths parametrize the Teichmüller space globally and showed the space is represented as a real analytic hypersurface of the parameter space.

(9) Representation of the mapping class group as a group of rational transformations (No. 24,25 and 26) : We proved that the mapping class group acts on a parameter space of the Teichmüller space as a group of rational transformations.

(10) Weil-Petersson geometry of Teichmüller spaces. We define Fenchel-Nielsen deformations of Fuchsian groups and reproved Wolpert's formula which concerns infinitesiomal variation of lengths of closed geodesic along a Fenchel-Nielsen deformationfor two-dimensional Teichmüller spaces. (No.27) :