Background Just as the tensor product of a representation and its irreducible decomposition are important in the representation theory, the notion of tensor product of a VOA module and its structure are important in vertex operator algebras. Since the definition of VOA modules is more complicated than the definition of representations of algebras, it is not easy to define the tensor product. For example, when a suitable product is defined for two VOA modules, it becomes very difficult to check whether the product is again contained in the category of the VOA modules, or whether the universality of the tensor product is satisfied. A series of papers by Huang, Lepowsky, and Zhang have shown that the category of logarithmic VOA modules admits the structure of a tensor category if some appropriate conditions are satisfied. One of the important concepts they use is the operator defined among modules, called the intertwining operators. The intertwining operators correspond to the chiral fields in conformal field theories and play important roles in studying the structure of the tensor products of VOA modules.

In the representation theory of vertex operator algebras, there is an explicit construction method of modules called free field realization. With this method, various vertex operators are explicitly realized using free fields of bosons and fermions. It is known that, under appropriate conditions, it is possible to define operators commutative with the action of VOA, called screening operators, between the modules constructed by the free field realization. Fjelstad et al. and Li, from the viewpoints of conformal field theory and vertex operator algebra, respectively, have found a method of logarithmic deformation of certain operators using screening operators (Fjelstad et al. *Phys*, 2002 & Li. *Algebra*, 1997). Their logarithmic deformation method allows us to construct nontrivial logarithmic modules from VOA modules bounded by screening operators. We expect that a suitable extension of the deformation method of Fjelstad et al. and Li will lead to deep results on the theory of intertwining operators.

Purpose and Contents In our previous works on triplet W-algebras and $N = 2$ Virasoro superalgebras, we found a certain ϵ deformation method for vertex operators (Nakano.,arXiv, 2024 & Nakano, Orosz Hunziker, Ros Camacho and Wood. arXiv, 2024). As mentioned in Past Research Results, this method can be called a coarse-graining of the representation by the small parameter ϵ, and an analytical evaluation on ϵ reveals properties of the intertwining operators and 4-point functions, which are important in the representation theory of VOA. We believe that this deformation technique is applicable not only to triplet W-algebras and Virasoro algebras but also to various vertex operator algebras. In this study, we will refine this deformation method and apply it to the study of the intertwining operators and N-point correlation functions of logarithmic vertex operator algebras.

In the theory of tensor categories, the associator, which appears in the definition of the Pentagon axiom, plays very important roles. In the formulation of Huang, Lepowsky, and Zhang, this operator is characterized by an intertwining operator satisfying a certain universality. If the properties of the associator is clarified, the structure of the tensor category is completely determined. Interestingly, by using the ϵ deformation technique in bosonic ghost VOA, Fjelstad et al. and Li's operator of logarithmic deformation can be derived, and the properties of the associator of the tensor category is also revealed (Allen, Lenther, Nakano, and Wood, in preparation). Here, bosonic ghost VOA is a vertex operator algebra, also called beta-gamma system, which is similar to and somewhat simpler than affine sl_2 VOA. In general, it is an extremely difficult problem to determine the properties of associators, but in this study, we attempt to approach this problem using the ϵ deformation method.

At the same time as us, Creutzig, McRae, and Yang obtained the same statements about the tensor category of $N = 2$ Virasoro superalgebra (see Past Research Results). Their method of proof differs from our analytic method and is more categorical. We will compare our method with theirs and pursue the theory of tensor categories of the underlying VOA modules.