Future research plans

[1] Computation and analysis of A type W-graphs using supercomputer.

For a Weyl group W, Kazhdan–Lusztig defined W-graph, which gives a representation space of W using certain data defined on a finite graph. In particular, for type A, so called the left cell representation is constructed using the Young tableaux. For the calculation, it is needed to calculate the recursively defined Kazhdan–Lusztig polynomials. When the order of the Weyl group is big, the calculation time and the resulting data become huge. By personal computer, I get the W-graph for the symmetric group of degree $n = 16$, $\lambda = (44422)$, $\dim = 171600$, which has edge multiplicities 1 or 5 only. To analyze the explicit graph structure of W-graphs, the data of higher degree n is needed. For the purpose, I plan to study by the following process. First store further calculation and store the data by parallel computing using high performance supercomputer with multi-core processor and large scale memory, Then analyze the data to extract the fundamental patterns and conjecture the general form to be proved. For the proof it will be helpful to compare the data with the theory of admissible W-graph by Stembridge. The induction-restriction theory of left cell will be also helpful. Recently, pattern analysis by using AI is performed actively. So, it will be also helpful for the consideration to use AI to search the behavior of the graph, when n becomes large.

[2] Springer representation and its relation to W-graph.

The Springer representation is known as a geometric realization of irreducible representation of Weyl group, but the explicit action on the basis coming from the irreducible components is only known as general formula. Even for type A case, the explicit form of the action in terms of Young tableaux is not known. The general formula is very similar to the W-graph action, and for some special cases, the two actions coincide. But in general, it is unclear whether it is always the case or some minor modification might be needed. Actually, to check the explicit calculation is difficult even for small example, and it is the key point how to extract combinatorial data form the geometric information. Related to [1], it will be expected to search for clues from the view point of analogy.

[3] Schubert calculus of p-compact group associated to the complex reflection group $G(r, 1, n)$.

Using equivariant version of Hall–Littlewood functions, we can consider Schubert like basis for equivariant chohomology of the Grassmannian space of p-compact group associated to the complex reflection group $G(r, 1, n)$. With respect to the classical flag varieties, I will try to consider the ring structure, such as Chevalley rule or combinatorial formula for the structure constants, by calculating explicit examples or referring to the non-equivariant formula. For the full flag case, there will be also a Schubert like basis with similar properties to be established, which I would like to investigate.

[4] Generalization of Nakada's hook formula using motivic Chern classes.

K-theory version of Nakada's hook formula using motivic Chern classes is already formulated, but it is a little complicated. In some cases, there are some simplified combinatorial formulae. Combinatorial objects such as the d-complete posets and the minuscule heaps are important for a generalization of Nakada's hook formula. Using these combinatorial structures effectively it will be expected to generalize the formula to the Kac–Moody Weyl group case. Elliptic analogue should be also considered.