Research Achievements

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My research focuses on the mathematics related to two-dimensional conformal field theory, and I have studied vertex algebras and factorization algebras.

Research 1: Operads of Supersymmetric Vertex Algebras

Vertex algebras are infinite-dimensional algebraic structures introduced by Borcherds in connection with the proof of the Moonshine conjecture, and are now known as an algebraic formulation of two-dimensional conformal field theory. They can be regarded as algebraic structures equipped with countably many operations indexed by integers $n \in \mathbb{Z}$, and thus have a more intricate structure than associative algebras or Lie algebras.

Operations in algebraic structures may satisfy symmetries such as associativity and commutativity, and operads provide a general framework to describe such symmetries. For example, giving a Lie algebra structure (a Lie bracket) on a vector space V is equivalent to giving a morphism from the Lie operad $\mathcal{L}ie$ to the endomorphism operad $\mathcal{E}nd_V$. In this sense, $\mathcal{L}ie$ is an operad that encodes the structure of Lie algebras. One advantage of describing algebraic structures via operads is that it allows one to introduce a dg Lie algebra that governs the deformation theory of the given algebra. An operad describing the structure of vertex algebras was constructed by Bakalov, De Sole, Heluani, and Kac.

On the other hand, in order to provide a unified treatment of superfields appearing in two-dimensional conformal field theories with supersymmetry, Heluani and Kac introduced the notion of supersymmetric vertex algebras (SUSY vertex algebras), which are analogues of vertex algebras in the superfield setting. In [NY25], we constructed an operad describing the structure of SUSY vertex algebras as an extension of the operad introduced by Bakalov et al., and computed the cohomology associated with deformation problems of SUSY vertex algebras. Furthermore, we constructed an operad for SUSY Poisson vertex algebras (SUSY Poisson vertex algebras), which arise as degenerations of SUSY vertex algebras, and investigated the relationship between these two operads.

Research 2: Vertex Algebras and Factorization Algebras on the Complex Plane

Factorization algebras, introduced by Costello and Gwilliam, are a concept obtained by abstracting the structure carried by spaces of observables in general quantum field theory. They constructed spaces of observables from perturbative quantum field theories on Riemannian manifolds, defined mathematically by Costello, and showed that these form a factorization algebra.

Since vertex algebras provide an algebraic formulation of two-dimensional conformal field theory, it is natural to expect a relationship between vertex algebras and factorization algebras on the complex plane $\mathbb{C}$. Indeed, Costello and Gwilliam provided a general method to extract a vertex algebra from a factorization algebra on $\mathbb{C}$. Moreover, they constructed examples of factorization algebras to which this method applies, starting from the affine vertex algebra and the $\beta\gamma$ vertex algebra, and showed that the resulting vertex algebras are isomorphic to the original ones. These factorization algebras are constructed using an important technique called factorization envelopes.

There is a class of vertex algebras called enveloping vertex algebras of Lie conformal algebras, which enjoy a universality property analogous to that of universal enveloping algebras of Lie algebras. Since the affine vertex algebra and the $\beta\gamma$ vertex algebra can be expressed as such enveloping vertex algebras, it is natural to expect that the construction of factorization algebras by Costello and Gwilliam can be generalized to this setting. In the preprint [N], we carried out this generalization. More precisely, given a Lie conformal algebra L, we constructed a factorization algebra via factorization envelope and showed that the associated vertex algebra is isomorphic to the enveloping vertex algebra of L. Furthermore, we extended this construction to the case where L is equipped with a parity grading (a $\mathbb{Z}/2\mathbb{Z}$-grading), obtaining new examples of factorization algebras corresponding to the Neveu–Schwarz vertex superalgebra and the $N = 2, 4$ vertex superalgebras.

[NY25] Y. Nishinaka, S. Yanagida, *Algebraic operads of SUSY vertex algebras and SUSY Poisson vertex algebras*, Adv. Math. **483** (2025) 110671, 115pp.

[N] Y. Nishinaka, *Factorization envelopes and enveloping vertex algebras*, preprint, arXiv: 2512.07635.