My current research plan is composed of the following six main projects. These are inspired by the successful use of support varieties across Hochschild, Hopf, and group cohomologies, which have marked numerous aspects of the modern algebraic and geometric treatment of representation theory. I am particularly interested in unifying the work of Boe–Kujawa–Nakano, Negron–Pevtsova, and Duflo–Serganova [14, 15, 16, 25, 52] with the categorical viewpoints introduced by Balmer [6, 7].

Project 1: Support for relative Hochschild cohomology. The deformation theoretical nature of twisted tensor products hints to the viability of a well-behaved relative support theory that would satisfy the tensor product property. Investigating this phenomenon would be the natural continuation of my thesis's work on relative Hochschild cohomology, and is deeply connected with *Project 2*. My research program includes expanding upon my findings in the relative case [55, 56] to develop a support theory in the spirit of Balmer's tensor-triangular geometry [8]. Ongoing research with Kujawa featuring this support theory reconciles the work of Boe–Kujawa–Nakano and Duflo–Serganova [14, 25].

Project 2: Finite generation of relative Hochschild cohomology. Relative Hochschild cohomology is defined by considering a subalgebra inside of an algebra. A support theory for relative Hochschild cohomology requires that such a pair algebra-subalgebra has a finitely generated cohomology, since this property enables the use of powerful geometric techniques. This question is intimately tied with knowing when the usual and the relative cohomologies coincide. Having provided a formalization of several equivalent ways of computing the graded algebra structure of the relative Hochschild cohomology in [56], I am uniquely positioned for this study on classical Lie superalgebras [49].

Project 3: Recovering Hochschild cohomologies from the relative setup. The recent discoveries and interactions between the usual and relative Hochschild cohomology [22, 46], unequivocally suggest that comprehending the relative setup provides valuable structural insights of the usual one [20, 21]. I want to establish conditions for when the relative Hochschild cohomology of a pair of quiver algebras reduces to the usual one. This will have important implications when considering finite generation questions, potentially providing examples that yield both finitely generated and non-finitely generated cohomologies.

Project 4: Invariance of relative Hochschild cohomology under derived equivalence. It is well known that Hochschild homology and cohomology are preserved under derived equivalence as a graded space and graded algebra with the cup product. Armenta and Keller recently proved that such a derived invariance also holds for the cap product [3, 61]. Conversations with Armenta, Cibils, and Keller suggested that translating these questions to the relative setup may yield positive answers, and looking into a "relative" derived invariance for a pair of algebras akin to a Morita equivalence would shed light on their representations.

Project 5: Hochschild cohomology of twisted planes. Interested in the Koszulity and Artin-Schelter regularity of quadratic twisted tensor products, Conner and Goetz [18] started what Bances and Valqui [10] developed into a nearly complete classification. In conjunction with Valqui and through the techniques I developed in [45, 58], I am completing this classification. It is sensible to expect that some of the unclassified twisted planes are rigid and have non-vanishing second Hochschild cohomology. These would be the first such examples that are found in characteristic zero.

Project 6: Classes of algebras preserved by twisted tensor products. Given a noncommutative construction, it is a classical problem to determine which properties it preserves [41]. Oswald and I proved in [59] that twisted tensor products of Hopf algebras never inherit a Hopf algebra structure, while twisted tensor products of Frobenius algebras always inherit a Frobenius algebra structure, and used this to prove that quantum complete intersections are noncommutative graded isolated singularities. Ongoing work in this direction is yielding additional applications.

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