My research is in *noncommutative algebra and noncommutative geometry*, where I use representation theory and homological tools to study quantum symmetries. A *classical* symmetry of an object is encoded by the action of a group. A *quantum* symmetry is given by the action of a Hopf algebra. These algebras not only play a key role throughout mathematics, they also enjoy deep connections to mathematical physics and applications to quantum computers.

I am particularly interested in algebras arising as *twisted tensor products*. This is a universal construction generalizing tensor product of algebras to the noncommutative setting. It encompasses vast families of algebras such as semidirect products, Drinfeld doubles, Weyl algebras, quantum elementary abelian groups, and quantum complete intersections. A major advantage it provides is the explicit description of large algebras in terms of smaller pieces. The idea is then as old as mathematics themselves; understand the whole by understanding the parts. A recurring theme throughout my work is the development of tools and techniques to handle twisted tensor products, shedding light on the categories of modules of the aforementioned Hopf and Frobenius algebras, with the end goal of understanding quantum symmetries.

To this effect, I first studied their *Hochschild cohomology*. Together with Karadag, McPhate, Oke, and Witherspoon, I provided an explicit formula to compute the cohomological Lie bracket in full generality [45]. These techniques recovered and expanded known results for skew group algebras, some quantum complete intersections, and the Jordan plane computations of Lopes–Solotar. In subsequent work, Oke, Witherspoon, and I [58] used homotopy liftings to recover Le–Zhou's isomorphism of Gerstenhaber algebras between the Hochschild cohomology of a tensor product and the tensor product of their respective Hochschild cohomologies, and Grimley–Nguyen–Witherspoon's analogous isomorphism on the bigraded Hochschild cohomology when twisting by a bicharacter.

My dissertation then focused on foundational aspects of relative homological algebra because the native setting to study the cohomology of twisted tensor products is, in fact, *relative Hochschild cohomology*. I proved a relative Künneth theorem [55], and used it to construct a cup product and Lie bracket yielding a Gerstenhaber algebra [56]. Establishing this formalism completed the fundamental work of Gerstenhaber and Schack to study deformations of algebras and their rigidity.

Recently Oswald and I characterized when the twisted tensor product of Hopf and Frobenius algebras inherits these structures [59]. Not only we recovered quantum complete intersections studied by Bergh–Erdmann, we also constructed infinite families of noncommutative symmetric Frobenius algebras. This was useful to notice that the canonical equivalences of (extended) commutative Frobenius algebras and 2-dimensional (unoriented) quantum field theories are symmetric monoidal [57]. After Balmer and I established a universal support theory for triangulated categories [9] parallel to Balmer's original spectrum, I want to understand these supports for the stable module categories of Frobenius and Hopf algebras coming from twisted tensor products.

This interest made me delve into combinatorics and homotopy theory. In both of these areas, the lattices that play a fundamental role in the universal support theory for triangulated categories also enjoy a predominant role. Seeking a deeper understanding of these relations led me to establish structural results for the tableau algebra [24] together with Daugherty, Gonzalez, Muniz, Pan, and Torres. Our results include showing that it is a flat degeneration of the partial flag variety. Investigating the combinatorial characterizations of N_∞-operads, together with Chan, Cho, Mehrle, Osorno, Szczesny, and Verdugo, I proved that a pairing of operads induces a pairing on the associated indexing systems. To show a partial converse, we then developed constructions useful to build compatible pairs of indexing systems from pairings of operads [19].

My research also concerns real-life applications. Together with Aduddell, Fairbanks, Kumar, Patterson, and Shapiro, I rigorously formalized the intuitive notion of a motif and their role as generators of regulatory networks, described functorial relations between regulatory networks and reaction networks, and provided the dynamics of regulatory networks in terms of Lotka–Volterra systems of ordinary differential equations [1]. Together with Aguilar, Álvarez, Ardila, Rodriguez Avila, and Várilly-Alvarado, I constructed infinite families of codes with desirable properties for applications to cloud storage: They correct a large number of errors while minimizing redundancy and overhead, and they maximize the availability of the user data while minimizing bandwidth usage [2].

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