Future Research Plan

My research aims to develop a unified framework for hypergeometric functions arising from those defined via integrals over quiver varieties. The goal of this project is to establish a systematic method for deriving global structures— such as differential and q-difference equations— from wall-crossing formulas, and to extend this framework to broader geometric and algebraic settings.

1. From Wall-Crossing to Differential/Deifference Equations

   - I will refine the Nakajima– Yoshioka blowup formula using wall-crossing techniques and construct commuting differential operators associated with partition functions,
   - I will extend the framework to affine type A systems, including partition functions arising from surface singularities and parabolic bundles,
   - I will analyze wall-crossing phenomena associated with imaginary roots using algebraic and analytic methods, incorporating Hodge-theoretic techniques and Hall algebra structures.

2. Generalization of Geometric and Cohomological Frameworks

   - I will extend wall-crossing theory beyond quiver varieties to a wider class of moduli spaces, including flag varieties, instanton moduli spaces, bow varieties, and moduli over weighted projective lines.
   - In parallel, I will reformulate wall-crossing phenomena using generalized cohomology theories such as elliptic cohomology, quantum cohomology, and quantum K-theory. This will provide a conceptual framework connecting geometry with representation theory and special functions.

3. Applications and Connections

   Building on this framework, I will explore applications to several advanced topics:

   - representation-theoretic interpretations of special functions via quantum toroidal algebras,
   - connections with vertex operator algebras through chiralization,
   - relations to 3-dimensional mirror symmetry, R-matrices and vertex functions.

Through these directions, I aim to position wall-crossing as a unifying principle linking geometry, representation theory, and special functions, and to uncover new structures in modern mathematical physics.