Research Plan (Yoshihiro OHNITA)

In this project we will intensively and broadly study symmetry, stability and moduli in submanifold geometry and harmonic map theory.

Submanifold geometry related to isoparametric submanifolds. Submanifold theory of Riemannian manifolds, which is a higher dimensional generalisation for the theory of curves and surfaces in Euclidean spaces, originating from Gauss and Riemann, is the mainstay of differential geometry. Submanifold geometry in symmetric spaces, including space forms of constant curvatures, is a rich and important discipline with a long tradition and history. When a submanifold is obtained as an orbit under a Lie subgroup of the isometry group, it is called *homogeneous*. In particular, in recent years the geometric structures for symmetric spaces and associated finite and infinite dimensional homogeneous spaces (e.g. R-spaces, generalized flag manifolds and k-symmetric spaces, loop groups and infinite dimensional Grassmannian models, Kac-Moody groups and Kac-Moody symmetric spaces etc.) and related submanifold geometry and harmonic map theory are highly advanced. Now the concept of *isoparametric hypersurfaces*, whose classification problem was first systematically studied by Elie Cartan, has also been extended and generalized to *isoparametric submanifolds* of general codimension and infinite dimension. Isoparametric submanifolds and their focal submanifolds are beautiful submanifolds with high symmetry. They are fundamental and endlessly interesting geometrical objects. In submanifold geometry in symmetric spaces, from the viewpoint of isoparametric submanifolds, it is quite interesting to explore the relationship among theories of submanifolds of different types and to find new properties, concrete examples and classifications.

Harmonic maps and relevant integrable systems Now it is a well-known fact that the harmomic map equation (HME) of a Riemann surface or a Lorentzian surface has the zero curvature formulation with a spectral parameter. This is an extremely beautiful conformity between the harmonicity of the map and the symmetry of the space. This fact enables us to treat HME as integrable systems and the first important paper is due to K. Uhlenbeck (JDG1989). For such harmonic maps, infinte dimensional loop group actions, infinte dimensional Weierstrass type formulas (DPW), gauge-theoretic equations, structures of moduli spaces and so on have been studied (Guest-Ohnita [21] etc.). It is also interesting to extend the study of harmonic maps of Riemann surfaces to pluriharmonic maps of higher dimensional complex manifolds (Ohnita-Valli [18] etc.). The integrable system structure of harmonic maps via loop group theory based on infinite dimensional Grassmannian models is not yet fully explored. Recently relevant oversea researchers make progress in the structural research on harmonic maps of finite uniton number over infinite dimensional Grassmannian models. This project aims to study a theory that goes beyond it. Also, inspired by N. J. Hitchin's work (JDG1990) on harmonic tori into S^3, around 2000 we have discussed the structure and geometry of the moduli spaces of solutions to the Yang-Mills-Higgs equations over Riemann surfaces, which were gauge-theoretically formulated from HME in compact Lie groups and symmetric spaces (M. Mukai-Hidano, Ohnita [32] etc.). Related research on this subject (applications of the Higgs bundle moduli spaces to differential geometry) has recently been intensified by several younger oversea researchers. I will continue to study harmonic maps by the method of Higgs bundles.

The above plans will be promoted by utilising the functions of OCAMI as a centre of excellence and by building links with other oversea and domestic research institutions, thereby contributing to the development of OCAMI.