（２） Future Research Plan

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Backgrounds to Research Plans

As already mentioned in the "Summary of Previous Research", it is important to study spin-3/2 Rarita–Schwinger fields on manifolds admitting real Killing spinors. Such manifolds are Einstein with positive scalar curvature, and were classified by C. Bär: they are either Sasaki–Einstein manifolds, 3-Sasakian manifolds, nearly Kähler manifolds, or nearly parallel G_2-manifolds. In the peer-reviewed papers [1] and [2], Rarita-Schwinger fields on nearly Kähler manifolds and on nearly parallel G_2-manifolds were fully characterized. It is therefore highly interesting to investigate Rarita-Schwinger fields on the remaining two classes—Sasaki–Einstein manifolds and 3-Sasakian manifolds.

Research Plans

(1) **Study Rarita-Schwinger fields on Sasaki-Einstein manifolds**

Recently, U. Semmelmann, C. Wang, and M.-Y. Wang obtained results on the linear stability of Sasaki–Einstein manifolds. Linear stability of an Einstein metric is defined in terms of the second variation of the total scalar curvature functional in transverse-traceless (TT) directions being negative. Their work shows that a Sasaki–Einstein manifold becomes linearly unstable when certain conditions on its Betti numbers are satisfied. The key idea is to compare the Laplacian associated with a "good connection" on a Sasaki manifold with the Laplacian of the Levi–Civita connection, and to verify instability by rewriting harmonic forms via this comparison.

I plan to adapt this framework to the Rarita–Schwinger operator: by expressing Rarita-Schwinger fields in terms of more tractable tensorial quantities, I aim to analyze their existence, their dependence on dimension, and their relationship to deformation theory. I am currently carrying out explicit computations in the case of 5-dimensional Sasaki–Einstein manifolds.

(2) **Study Rarita-Schwinger fields on 3-Sasakian manifolds**

A 3-Sasakian manifold is a special case of a Sasaki–Einstein manifold. Moreover, 3-Sasakian manifolds admit a "good connection" that does not exist on general Sasaki manifolds. Therefore, building on the approach in (1) and exploiting the additional symmetries, I aim to obtain an explicit description of Rarita-Schwinger fields and to establish a computable criterion for their existence.

(3) **A unified approach to spin-3/2 Rarita-Schwinger fields via Riemannian cone construction**

As the third pillar of my research plan, I aim to develop a framework that provides a unified description of Rarita–Schwinger fields on manifolds admitting real Killing spinors via their Riemannian cone constructions. More specifically, I will derive explicit relations between the Rarita–Schwinger operator on a manifold M and the corresponding operator on its Riemannian cone $\bar{M}$, and use these relations to reformulate the construction and analysis of Rarita–Schwinger fields in a more uniform manner, potentially within the scope of integrable geometric structures. Unlike the conventional approach, which typically constructs Rarita–Schwinger fields case by case on individual manifolds, this strategy seeks a common mechanism and a theoretical unification of the relevant operators across the entire class of manifolds admitting real Killing spinors. I expect that such a unified perspective will open new avenues both in geometry and in the mathematical foundations of higher-spin field theory.

Last year, I spent approximately three weeks at the University of Stuttgart, where I discussed these topics with U. Semmelmann. I plan to continue developing this line of research through sustained collaboration and ongoing discussions.

(4) **Study invariant Rarita-Schwinger fields on nilpotent manifolds**

Recently, G. Bazzoni, L. Martín-Merchán, and V. Muñoz investigated harmonic spinors and eigenvalues of the Dirac operator on nilmanifolds. Their approach relies on the fact that the Dirac operator acting on left-invariant spinors can be written in an explicit and particularly tractable form, and they then carry out case-by-case computations for concrete nilmanifolds in each dimension using available classification results. I expect that this strategy can be adapted to the Rarita–Schwinger operator as well. In the longer term, I aim to study (left-invariant) Rarita-Schwinger fields on a broader range of Lie groups.